Governing Interest Groups and Rent Dissipation

by

Gil S. Epstein
Bar-Ilan University, Israel and CReAM, London, IZA, Bonn

and

Yosef Mealem
Netanya Academic College, Netanya, Israel

Abstract

In a contest group - specific public goods we consider the effect that managing an interest group has on the rent dissipation and the total expected payoffs of the contest. While in the first group, there is a central planner determining its members’ expenditure in the contest, in the second group there are two different possibilities: either all the members are governed by a central planner or they aren’t. We consider both types of contests: an all pay auction and a Logit contest success function. We show that while governing an interest group decreases free-riding, it may as well decrease the rent dissipation; at the same time the expected payoffs from the groups may also decrease.

Keywords: Rent dissipation, Central planner, Contest, All pay auction, Logit contest success function.

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1. Introduction

Government intervention often gives rise to contests in which the possible prizes are determined by the status-quo and some new public-policy proposals. Since a proposed policy reform has different implications for different interest groups, the groups make every effort to influence the approval of the proposed public policy makers in their favor. A major concern in the contest literature has been the issue of how changes in the parameters of the contest (number of the players, valuations and abilities of the contestants and the nature of the information they have) alter the equilibrium efforts and the extent of relative prize dissipation: see Hillman and Riley (1989), Hurley and Shogren (1998), Konrad (2002), Nitzan (1994) and Nti (1997). In addition, attention is paid to the effect of the changes made in these parameters on the contestants’ expected payoffs, as in Gradstein (1995) and Nti (1997). Moreover, a major theoretical effort has been made to clarify the different levels of rent under-dissipation in the contests, Gradstein and Konrad (1999), Kahana and Nitzan (1999), Konrad (2004), Konrad and Schlesinger (1997), Nitzan (1994), Nti (1997). However Epstein, Nitzan and Schwarz (2009) show that while contestants usually expend resources in trying to win the contested prize, potential recipients of the rent-seeking efforts also participate in the contest. This is due to an uncertainty regarding the source of power in the contest as a result of which rent dissipation may well increase.

In this paper the case of a two interest group is considered. We compare two situations. In the first, one group is governed by a central planner and the other isn't, and in the second, both groups are governed by central planners. Our objective is to compare the rent dissipation and the payoffs in the two different contests. For example, consider the case of a firm that is defending its dominant power over consumers who are challenging it: Baik (1999), Ellingsen (1991), Epstein and Nitzan (2003, 2007) and Schmidt (1992). In the first case, the firm is one entity, while the consumers may or may not be organized. We examine the effect of consumers who are organized as one group, and governed by a central planner comparing them to a situation where the consumers are not organized. It is clear that the consumers, while organized, will decrease the free-riding which would occur without a central planner; however, it is not clear how organizing the consumers under a central planner will affect the firm’s efforts in the contest. One could think of many other situations in which this could occur, such as the struggle over the determination for minimum
wages, migration quotas etc. This comparison is carried out considering two different contest success functions: 1. The generalized logit function; 2. The All Pay Auction.

Riaz, Shogren and Johnson (1995) deal with a general model of rent seeking for the public good. In the paper they demonstrate that total effort expended in a collective rent-seeking contest depends on the group size. Moreover, they show that rent seeking increases as the group size increases. In our paper we take a different approach. Considering the situation of a given group's size, the question we wish to analyze is whether letting a group to be governed by a central planner will increase or decrease total expenditure in the contests, and what will be the effect on a total expected payoff.

Although the results we obtain for both types of contest success functions are quite similar, they do not always coincide. Under the generalized Logit contest success function, when a group becomes organized, its expenditure increases, but it is not clear what will happen to its expected payoff. On the other hand, the opponents' expected payoff decreases while it is not clear what will happen to its expenditure in the contest. Surprisingly, when a group becomes organized, it may well be that both the rent dissipation in the contest and the total expected payoff will increase.

Under the all pay auction contest success function, when a group becomes organized, its expenditure does not decrease, yet it is not clear what happens to its expected payoff. The opponents' expected payoff will not increase, while it is not clear what will happen to its expenditure. As in the results obtained in the logit contest success function, there are situations under which both the rent dissipation and the total expected payoffs increase (even though the conditions are not identical to those of the logit function). This emphasizes the result presented by Hurley (1998) who has shown that rent dissipation can be a misleading measure of the welfare implications of a contest when players have asymmetric valuations. The results obtained depend on deciding which of the interest groups is the favorite, and which is the underdog. This issue will be further elaborated on below.

In addition, we show that the rent dissipation may decrease in such a situation. The intuition for the decrease in the rent dissipation is that if the parties are strategically symmetric in the absence of coordinated action inside the group which constitutes one party, coordination inside the group makes this group much stronger, and so generates the asymmetry which reduces the amount of the overall rent seeking in the contest.
Examples of such situations would be the struggle over a monopoly when there are a large number of consumers, each with a low benefit from winning the contest or workers who are struggling for an increase in minimum wages. We show that under the logit contest success function governing this group will increase the probability of the group winning the contest, and at the same time decrease the rent dissipation and the effect on the total expected payoffs, which may also increase as a result. This, in turn, will increase welfare.

2. The Model

Consider a contest with two groups competing for a prize. As in Epstein and Nitzan (2007), suppose that a status-quo policy is challenged by one interest group, and defended by the other. This policy can be the price of a regulated monopoly, the maximal degree of pollution the government allows, the existing tax structure, the determination of the minimum wage, etc. The defender of the status-quo policy (henceforth, the defending interest group) prefers the status-quo policy to any alternative policy. The challenger of the status-quo policy (the challenging group) prefers the alternative strategy. For example, in the contest over monopoly regulations studied in Baik (1999), Ellingsen (1991), Epstein and Nitzan (2003, 2007) and Schmidt (1992), the firm with the monopoly defends the status-quo by lobbying for the profit-maximizing monopoly price (and against any price regulation), while consumers challenge the status-quo lobbying effort, preferring a competitive price. In a different example, the defending group would be the employers endorsing the status-quo against the worker's union that wants to change the minimum wage (Epstein and Nitzan, 2006b, 2007). In the challenging group (the consumers and the employees), there are $N$ players, and in the defending group there is one player (the monopoly, the capital owners). In the challenging group each player receives a benefit of $n_i$ from winning the contest, and the defender (the monopoly) receives a benefit of $m$. Player $i$ ($i = 1, \ldots, N$) invests $x_i$ resources to change the status-quo, and the defending player invests $y$ units in the contest.

We consider two types of contests: 1. The generalized logit function; 2. The All Pay Auction. We start by considering the case of the generalized logit contest success function, and then analyze the situation under the all pay auction contest success function.
2.1 The Generalized Logit Contest Success Function

2.1.1 An unorganized group

The probability that the new policy will be accepted and the status-quo changed, \( \Pr_N \), is given by the generalized logit contest success function as (Tullock, 1980):

\[
\Pr_N = \frac{\sum_{i=1}^{N} x_i^\alpha}{\sum_{i=1}^{N} x_i^\alpha + y^\alpha} \quad \text{with} \quad 0 < \alpha < 1 \quad (1)
\]

We restrict our analysis to the case where \( 0 < \alpha < 1 \) (see Epstein and Mealem, 2009).

The expected payoff of each of the players in the challenging group equals:

\[
E(U_i) = \frac{\sum_{i=1}^{N} x_i^\alpha}{\sum_{i=1}^{N} x_i^\alpha + y^\alpha} (n - x_i) \quad i = 1, \ldots, N \quad (2)
\]

and the expected payoff of the defining group equals:

\[
E(U_d) = \frac{y^\alpha}{\sum_{i=1}^{N} x_i^\alpha + y^\alpha} m - y \quad (3)
\]

Defining the ratio of the stakes between the challenging group’s stake and the defender's stake by \( k: k = \frac{n}{m} \). Solving the first order conditions for each of the players in the different groups (it can be verified that the second order conditions hold), we obtain that the investments in equilibrium, \( (x_i, y) \) and the probability of the challenging group winning the contest equal:

\[
x_i^* = \frac{\alpha n^{1\alpha} m^\alpha}{N^{\alpha} \left(N^{1\alpha} n^\alpha + m^\alpha\right)^2}, \quad y^* = \frac{\alpha N^{1\alpha} m^{1\alpha} n^\alpha}{\left(N^{1\alpha} n^\alpha + m^\alpha\right)^2}
\]

and

\[
\Pr^*_N = \frac{N^{1\alpha} n^\alpha}{N^{1\alpha} n^\alpha + m^\alpha} = \frac{N^{1\alpha} k^\alpha}{N^{1\alpha} k^\alpha + 1} \quad (4)
\]
The expected payoffs of the players equal:

\[
E(U_i^*) = \frac{n^{1-\alpha} \left[ N^{2-\alpha} n^\alpha + m^\alpha (N - \alpha) \right]}{N^{\alpha} \left( N^{1-\alpha} n^\alpha + m^\alpha \right)^2}
\]

and

\[
E(U_d^*) = \frac{m^{1+\alpha} \left[ m^\alpha + N^{1-\alpha} n^\alpha (1 - \alpha) \right]}{\left( N^{1-\alpha} n^\alpha + m^\alpha \right)^2}
\]  

Using (4) we can calculate the rent dissipation (RD) in the contest and obtain:

\[
RD^* = N x_i^* + y^* = \frac{\alpha N^{1-\alpha} n^\alpha m^\alpha (n + m)}{\left( N^{1-\alpha} n^\alpha + m^\alpha \right)^2}
\]  

and the total expected payoffs of the players equal:

\[
\sum E(U_i^*) + E(U_d^*) = \frac{N^{1-\alpha} n^{1+\alpha} \left[ N^{2-\alpha} n^\alpha + m^\alpha (N - \alpha) \right] + m^{1+\alpha} \left[ m^\alpha + N^{1-\alpha} n^\alpha (1 - \alpha) \right]}{\left( N^{1-\alpha} n^\alpha + m^\alpha \right)^2}
\]

2.1.2 The challenging group is governed by a central planner

Assume that the challenging group is governed by a central planner who determines the investment of each of the players in the group. The expected payoff of the challenging group becomes:

\[
\sum_{i=1}^{N} E(U_i) = \frac{\sum_{i=1}^{N} x_i^\alpha}{\sum_{i=1}^{N} x_i^\alpha + y^\alpha} \cdot N n - \sum_{i=1}^{N} x_i
\]

while expected payoff of the defending group is defined by (3).
Solving the first order conditions for each player in the different groups (the second order conditions hold) we obtain that the investments in equilibrium, \(( x, y)\) and the probability of the challenging group winning the contest equal:

\[ x_i^* = \frac{\alpha N n^{1+\alpha} m^\alpha}{\left( N n^{\alpha} + m^\alpha \right)^2}, \quad y^* = \frac{\alpha N n^{\alpha} m^{1+\alpha}}{\left( N n^{\alpha} + m^\alpha \right)^2} \]

and

\[ \Pr_{N}^* = \frac{N n^{\alpha}}{N n^{\alpha} + m^\alpha} = \frac{N k^\alpha}{N k^\alpha + 1} \] \hspace{1cm} (9)

The expected payoffs of the players equal:

\[ E(U_i^{**}) = \frac{N n^{1+\alpha} \left[ N n^{\alpha} + m^\alpha (1-\alpha) \right]}{\left( N n^{\alpha} + m^\alpha \right)^2} \]

and

\[ E(U_d^{**}) = \frac{m^{1+\alpha} \left[ m^\alpha + N n^\alpha (1-\alpha) \right]}{\left( N n^{\alpha} + m^\alpha \right)^2} \] \hspace{1cm} (10)

Using (9) the rent dissipation \((RD)\) in the contest equals:

\[ RD^{**} = N x_i^* + y^* = \frac{\alpha N n^{\alpha} m^\alpha (N n + m)}{\left( N n^{\alpha} + m^\alpha \right)^2} \] \hspace{1cm} (11)

and the total expected payoffs of the players equal:

\[ \sum E(U_i^{**}) + E(U_d^{**}) = \frac{N^2 n^{1+\alpha} \left[ N n^{\alpha} + m^\alpha (1-\alpha) \right] + m^{1+\alpha} \left[ m^\alpha + N n^\alpha (1-\alpha) \right]}{\left( N n^{\alpha} + m^\alpha \right)^2} \] \hspace{1cm} (12)

2.1.3 Comparing the two situations
In this section we wish to compare the two cases. In the first, each of the players in the challenging group invests optimally in the contests and in the other, while the challenging group is governed by a central planner who determines the amount that each player invests.
Let us start by considering the total investment made by the challenging group. We wish to see if a central planner will increase or decrease the investment of the group. Since all the players are identical, \( \sum_{i=1}^{N} x_i^{**} > \sum_{i=1}^{M} x_i^{*} \) holds if and only if \( x_i^{**} > x_i^{*} \). Thus, \( \sum_{i=1}^{N} x_i^{**} > \sum_{i=1}^{M} x_i^{*} \) if \( \sum_{i=1}^{N} x_i^{**} = \frac{\alpha N^2 n^1 \alpha m^a}{(Nn^a + m^a)^2} > \frac{\alpha N^{1-\alpha} n^1 \alpha m^a}{(N^{1-\alpha} n^a + m^a)^2} = \sum_{i=1}^{N} x_i^{*} \).

This is identical to \( N^a \left(N^{0.5-0.5a} - 1\right) > m^a \left(1 - N^{0.5+0.5a}\right) \), which always holds. Thus,

**Lemma 1:** The central planner increases the investment made by the group: \( \sum_{i=1}^{N} x_i^{**} > \sum_{i=1}^{M} x_i^{*} \), and increases the probability of winning \( Pr_N^{**} > Pr_N^{*} \).

The lemma states that a central planner will increase the total investment of the group relative to the same investment, if the group is not organized by the planner. The main idea behind this result states that the central planner can decrease free-riding, and thus increase the total investment of the groups.

Following Dixit (1987) and Riaz, Shorgren and Johnson (1995) define the contest “favorite” and “underdog”. The favorite is the group whose probability of winning at Nash equilibrium exceeds one-half whereas its rival is the underdog.

If the challenging group is not organized, it is the favorite if and only if \( N^{1-\alpha} k^a > 1 \). If there is a central planner, it is favorite if and only if \( N k^a > 1 \). In both cases a sufficient condition for the challenging group to be a favorite is \( k \geq 1 \). This is not surprising since when \( k \geq 1 \) the value of each of the players in the challenging group is higher than in the other group as well as the fact that number of players is also larger and thus we obtain that this group is the favorite. We will return to this later on in the paper.

Consider the investment made by the defending group. The defending group will invest less than would have been invested if the challenging group had not been governed by a central planner if \( y^{**} < y^{*} \). From (9) and (4) \( y^{**} < y^{*} \) if

\[
y^{**} = \frac{\alpha N n^1 \alpha n^a}{(Nn^a + m^a)^2} < \frac{\alpha N^{1-\alpha} n^1 \alpha n^a}{(N^{1-\alpha} n^a + m^a)^2} = y^{*}.
\]

This condition is equal to \( N > k^{\alpha/2} \) thus,
**Proposition 1**: The defending group will decrease its efforts as a result of the central planner governing the challenging group, \( y^* < y^* \).

1. if and only if \( N > k^{a-2} \).
2. A sufficient condition would be that the stake of a member of the challenging group will be at least as large as the stake of the defending group: \( n \geq m \) \((k \geq 1)\).

Proposition 1 states that it is not clear if the defending group invests more or less when the challenging group is organized. Once the challenging group increases its investment by becoming organized, the defending group may increase or decrease its investment.\(^2\) The condition which must hold depends on the ratio of the stakes and the number of players in the challenging group. If \( N > k^{a-2} \), then the defending group will decrease its investment when the challenging group becomes organized. Thus, the idea here would be that as the challenging group increases the total investment, it forces the defending group to decrease its expenditure. In the second part of the proposition, a sufficient condition is given to ensure the decrease of the defending group’s investment. This condition states that if the stake of a member of the challenging group is at least as large as that of the defending group's stake, then the defending group will decrease its investment. This means that the challenging group, as a group, has a lot more to gain from the contests. The organized group will increase its investment and force the defending group to decrease theirs. Moreover, the sufficient condition in the second part of the proposition, \( k \geq 1 \), is identical to stating that the organized group is the *favorite*. In other words, if the challenging group is the favorite, then becoming organized increases this group's efforts (lemma 1) making the group even stronger, and forcing the defending group to decrease its effort in the contests.

**Rent Dissipation (RD) and Expected Payoffs**

As we have seen, organizing the challenging group will increase its investment while it may or may not increase the defending group's investment. The question, which is

\(^2\) This result is somewhat similar to the result presented in Epstein and Nitzan (2006a) where they present a case in which increasing the stake of one party sufficiently may increase that group's investment, and cause the other group to decrease its investment.
now posed, is what happens to the total investment and to the expected payoffs of both groups. In other words, what happens to the rent dissipation of the contest and to the expected payoffs as a result of the challenging group becoming organized?

We would, therefore, like to see if a situation exists under which the total investment in the contest decreases. The rent dissipation is seen many times as wasteful resources invested in the contest. Decreasing the wasteful resources can often be an indicator for welfare enhancing. This would mean that the increase in the investment of the challenging group would be smaller than the decrease in the defending group's investment. Note that Hurley (1998) mentions that rent dissipation can be a misleading measure of the welfare implications of a contest when players have asymmetric valuations. Thus, we will also consider how the sum of expected payoffs for all the players compares with and without a central decision maker for the challenging group.

When the challenging group is unorganized, the $RD$ of the contest will be higher than when the challenging group is organized, $(RD^{**} < RD^{*})$ if $N^x + y^{**} < N^x + y^{*}$. This is identical to

$$N^x + y^{**} = \frac{\alpha N_m^a m^\alpha (Nn + m)}{(Nn^\alpha + m^\alpha)^2} < \frac{\alpha N_1^\alpha n^\alpha m^\alpha (n + m)}{(N_1^\alpha n^\alpha + m^\alpha)^2} = N^x + y^{*}.$$  

Substituting $k = \frac{n}{m}$, the condition becomes

$$\frac{N}{Nk + 1} - \left(\frac{N^k + N}{Nk + 1}\right)^2 > 0 \text{ or }$$

$$0 < k^{1+2\alpha} N^{2+\alpha} (1 - N^1) + 2k^{1+\alpha} N^{1+\alpha} (1 - N) + kN^\alpha (1 - N^{1+\alpha}) + (k^2 N^2 - N^\alpha) (N^\alpha - 1)$$

(13)

Define the RHS of (13) by $G$:

$$G = k^{1+2\alpha} N^{2+\alpha} (1 - N^1) + 2k^{1+\alpha} N^{1+\alpha} (1 - N) + kN^\alpha (1 - N^{1+\alpha}) + (k^2 N^2 - N^\alpha) (N^\alpha - 1)$$

Condition (13) may hold. For example, if $\alpha = 0.7$, $N = 20,000$ and $k = 0.003$, then $G > 0$ $(RD^{**} < RD^{*})$.

$G$ consists of 4 components. The first three components are all negative. Thus, in order for the total summation to be positive, the last component must be positive. This will be true if $k^2 N^2 - N^\alpha > 0$ is equivalent to $N > k^{2/\alpha - 2}$. This is, therefore, a
necessary but not a sufficient condition for (11) to hold, and not surprisingly, the necessary and the sufficient condition for $y^{**} < y^*$ (notice that according to lemma 1 it is always true that $\sum_{i=1}^{N} x_{i}^{**} > \sum_{i=1}^{M} x_{i}^{*}$; therefore, for $RD^{**} < RD^*$ it must be the case that $y^{**} < y^*$).

**Proposition 2:**

**A. Rent dissipation**

As a result of the challenging group becoming organized, the rent dissipation of the contest will:

1. increase if the stake of a member of the challenging group is at least as large as that of the defending group ($k \geq 1$).
2. decrease if the stake of a member of the challenging group is sufficiently small, while the number of players in this group is sufficiently large.

**B. Total expected payoffs**

As a result of the challenging group becoming organized, the total expected payoffs of the players will:

1. increase if the stake of a member of the challenging group is high enough.
2. decrease if the stake of a member of the challenging group is sufficiently small, specifically if $k \leq \frac{1 - \alpha}{N}$.

**C. Expected payoffs of each group**

As the challenging group becomes organized:

1. the expected payoffs of the defending group will decrease.
2. the expected payoffs of the challenging group will decrease if $k$ is small enough and $\alpha \rightarrow 1$, and will increase if $k$ is high enough.

For proof of proposition 2 see Appendix 1.

Proposition 2 part A1 states that if the challenging group is the favorite (this occurs when the stake of a member of the challenging group is larger than that of the defending group), then as a result of the challenging group's organizing, the rent dissipation of the contest will increase. **This result is independent of the number of**
player in the challenging group, and depends only on the size of the stake of a single player in the group. In other words, this means that in the case of \( k \geq 1 \), and as a result of the challenging group becoming organized, even though the defending group will decrease its effort, \( y'' < y' \) (proposition 1) the challenging group will increase its effort. Moreover, if the stake of a member of the challenging group is high enough (which makes the challenging group the favorite) the results of proposition 2 part A1 and B1 will be that the rent dissipation, and the expected payoffs of the contest will increase. The idea behind this result is that if the stakes are sufficiently high, and the asymmetry between the contestants is also high, the favorite group will have a bigger advantage over its opponent, the underdog.

Proposition 2 part A2 and B2 state that the rent dissipation and expected payoffs may decrease. This occurs when the challenging group is the underdog. Thus, when the challenging group becomes organized, it increases its expenditure, but with the decrease in expenditure of the defending group, the total investment in the contest will decrease. The intuition for this result is that, if the two parties are strategically symmetric, in the absence of coordinated action inside the challenging group, coordination inside the other group will make it much stronger, and this will generate the asymmetry, which reduces the amount of overall rent-seeking in the equilibrium. In this situation, we would have many players in the challenging group, (the underdog), all having low benefits from winning the contest. This means that when they become organized, the total investment of the group increases causing the defending group to decrease its expenditure resulting in lower rent dissipation. Note that in such a situation, the probability of the challenging group winning the contest will increase. In the case of the monopoly story, by organizing the consumers, the probability of their winning increases because of the lower cost to society, and due to the decrease in the total expenditure.

Note that when the challenging group becomes organized, the expected payoff of the defending group decreases; however, the challenging group's expected payoff may decrease too. This is because when the challenging group becomes organized the defending group changes its strategy, and even though the challenging group replies optimally, its payoff could be decreased.

As we have seen from proposition 2 part A1, if \( k \) is sufficiently high (and therefore the challenging group is the favorite), becoming organized increases the
total rent dissipation in the contest. However, at the same time the challenging
group’s expected payoff increases, (proposition 2 part C2) and the expected payoff of
the defending group decreases (this is always true according to proposition 2 part C1)
in such a way that the total expected payoff increases (proposition 2 part B1). A high
\(k\) increases the asymmetry between the contestants, and increases the advantage the
favorite group has. This coincides with the findings of Hurley (1998) who has shown
that rent dissipation can be a misleading measure of the welfare implications of a
contest, when the players have asymmetric valuations. In our situation both the rent
dissipation and the total expected payoffs increase. In this case, total expected payoffs
would be a better indicator for welfare affects.

2.2 The All-Pay Auction Contest Success Function

The all-pay auction contest success function is given by:

\[
\Pr_n\left(\sum_{i=1}^{N} x_i, y\right) = \begin{cases} 
1 & \text{if } \sum_{i=1}^{N} x_i > y \\
0.5 & \text{if } \sum_{i=1}^{N} x_i = y \\
0 & \text{if } \sum_{i=1}^{N} x_i < y 
\end{cases}
\]

At equilibrium, the only active player in the challenging group is the one whose
valuation for the prize is the highest and all the other players expend zero effort (see
Baik, Kim, and Na, 2001). Since all the players in the challenging group have the
same valuation, in equilibrium only one of the players in the group is active. We will
now compare the situation of an unorganized group to the case of an organized group
under three situations, 1. \(k \geq 1\), 2. \(k \leq \frac{1}{N}\) and 3. \(\frac{1}{N} < k < 1\) (Note that \(k = \frac{n}{m}\)).

2.2.1 An un-organized group

1. If \(k \geq 1\) (\(n \geq m\):

If the group is not organized, then:

\[13\]

3 The results presented use the standard techniques as presented by Baye, Kovenock and de Vries (1993) and Konrad (2004).
\[
E\left(\sum_{i=1}^{N} x_i^\ast\right) = \frac{m}{2}, \quad E(y^\ast) = \frac{m^2}{2n} \quad \text{and} \quad RD^\ast = E\left(\sum_{i=1}^{N} x_i^\ast + y^\ast\right) = \frac{m(n+m)}{2n}.
\] (15)

The winning probability of the challenging group is \(\Pr_N^\ast = 1 - \frac{m}{2n} = 1 - \frac{1}{2k}\). Since \(k \geq 1\), the challenging group is the favorite (\(\Pr_N^\ast \geq 0.5\)). Assuming that player 1 is the active player in the challenging group, the expected payoffs of the players equal
\[
E(U_i^\ast) = n - m, \quad E(U_1^\ast) = \left(1 - \frac{m}{2n}\right)n = n - \frac{m}{2} \quad \text{for} \quad i = 2, \ldots, N \quad \text{and} \quad E(U_d^\ast) = 0.
\]

2. If \(k < 1\) (\(n < m\)):

If the group is not organized, then:
\[
E\left(\sum_{i=1}^{N} x_i^\ast\right) = \frac{n^2}{2m}, \quad E(y^\ast) = \frac{n}{2} \quad \text{and} \quad RD^\ast = E\left(\sum_{i=1}^{N} x_i^\ast + y^\ast\right) = \frac{n(n+m)}{2m}.
\] (16)

The winning probability of the challenging group is \(\Pr_N^\ast = \frac{n}{2m} = \frac{k}{2}\). Since \(k < 1\), the challenging group is the underdog (\(\Pr_N^\ast < 0.5\)). Assuming that player 1 is the active player in the challenging group, the expected payoffs of the players equal \(E(U_i^\ast) = 0\),
\[
E(U_i^\ast) = \frac{n^2}{2m} - n = \frac{n^2}{2m} \quad \text{for} \quad i = 2, \ldots, N \quad \text{and} \quad E(U_d^\ast) = m - n.
\]

2.2.2 The challenging group is governed by a central planner

1. If \(\frac{1}{N} < k\) (\(nN > m\)):

If there is a central planner in the challenging group, the total value of the challenging group is \(nN\). Since \(nN > m\) therefore:
\[
E\left(\sum_{i=1}^{N} x_i^{**}\right) = \frac{m}{2}, \quad E(y^{**}) = \frac{m^2}{2nN} \quad \text{and} \quad RD^{**} = E\left(\sum_{i=1}^{N} x_i^{**} + y^{**}\right) = \frac{m(nN+m)}{2nN}.
\] (17)
The winning probability of the challenging group is $Pr_N^{**} = 1 - \frac{m}{2nN} = 1 - \frac{1}{2Nk}$. Since $k > \frac{1}{N}$, the challenging group is the favorite ($Pr_N^{**} > 0.5$). If the central planner divides the total expenditure equally among the players in the challenging group, then the expected payoffs of the players equal $E(U_i^{**}) = \frac{nN - m}{N} = n - \frac{m}{N}$ for $i = 1, \ldots, N$ and $E(U_d^{**}) = 0$.

2. If $k \leq \frac{1}{N}$ ($nN \leq m$):
If there is a central planner in the challenging group, the total value of the challenging group is $nN$. Since $nN \leq m$ therefore:

$$E\left(\sum_{i=1}^{N} x_i^{**}\right) = \frac{(nN)^2}{2m}, \quad E(y^{**}) = \frac{nN}{2} \quad \text{and} \quad RD^{**} = E\left(\sum_{i=1}^{N} x_i^{**} + y^{**}\right) = \frac{nN(nN + m)}{2m}. \quad (18)$$

The winning probability of the challenging group is $Pr_N^{**} = \frac{nN}{2m} = \frac{kN}{2}$. Since $k \leq \frac{1}{N}$ the challenging group is the underdog ($Pr_N^{**} \leq 0.5$). If the central planner divides the total expenditure equally among the players in the challenging group, then the expected payoffs of the players equal $E(U_i^{**}) = 0$ for $i = 1, \ldots, N$ and $E(U_d^{**}) = m - nN$.

2.2.3 Comparing the two situations
Let us compare the rent dissipation in both cases.

1. If $k \geq 1$ ($n \geq m$) therefore $k > \frac{1}{N}$ ($nN > m$):

We obtain that $E\left(\sum_{i=1}^{N} x_i^{*}\right) = \frac{m}{2} = E\left(\sum_{i=1}^{N} x_i^{**}\right)$, $E(y^{*}) = \frac{m^2}{2n} > \frac{m^2}{2nN} = E(y^{**})$, $\sum_{i=1}^{N} E(U_i^{*}) = nN - \frac{m(N + 1)}{2} < nN - m = \sum_{i=1}^{N} E(U_i^{**})$ and $E(U_d^{*}) = E(U_d^{**}) = 0$. Therefore $RD^{*} > RD^{**}$ and $\sum E(U_i^{*}) + E(U_d^{*}) < \sum E(U_i^{**}) + E(U_d^{**})$. 
2. If $k \leq \frac{1}{N}$ ($nN \leq m$) therefore $k < 1$ ($n < m$):

We obtain that $E\left(\sum_{i=1}^{N} x_{i}^{*}\right) = \frac{n^{2}}{2m} < \frac{(nN)^{2}}{2m} = E\left(\sum_{i=1}^{N} x_{i}^{**}\right)$, $E(y^{*}) = \frac{n}{2} < \frac{nN}{2} = E(y^{**})$,  
$$
\sum_{i=1}^{N} E(U_{i}^{*}) = \frac{n^{2}(N-1)}{2m} > 0 = \sum_{i=1}^{N} E(U_{i}^{**}) \quad \text{and} \quad E(U_{d}^{*}) = m - n > m - nN = E(U_{d}^{**}).
$$
Therefore $RD^{*} < RD^{**}$ and $\sum E(U_{i}^{*}) + E(U_{d}^{*}) > \sum E(U_{i}^{**}) + E(U_{d}^{**})$. These results are quite surprising since when the challenging group becomes organized, the rent dissipation increases and the expected payoff of each player decreases, except for the active player in the challenging group, whose payoff has not changed.

The logic behind these results is the following: When the challenging group is not organized it has a lower value and the "symmetry" of the contest (hereafter "the first contest") depends on the "gap" between the two values, $n$ and $m$. Given $m$ ($n$) the higher (lower) $n$ ($m$) the contest is more "symmetric," and as a result, the expenditure of both groups increases. Now let us see what would happen to the "symmetry" of the contest when the challenging group becomes organized. In this case the challenging group has a higher value but still less than $m$ ($nN < m$). Since the "gap" between the two values decreases, (moving from $n < m$ to $nN < m$) the contest becomes more "symmetric" in comparison to the first contest, which results in a rise of the total investment, and a reduction in the expected payoff.

3. If $\frac{1}{N} < k < 1$ ($n < m < nN$):

We obtain that $E\left(\sum_{i=1}^{N} x_{i}^{*}\right) = \frac{n^{2}}{2m} < \frac{m}{2} = E\left(\sum_{i=1}^{N} x_{i}^{**}\right)$ (since $k < 1$), 
$$
E(U_{d}^{*}) = m - n > 0 = E(U_{d}^{**}).
$$
In this case the rest of the results depend on the relationship between $k$ and $N$, while in the former cases the obtained results are independent of that relationship. When the challenging group becomes organized, the expenditure of the defending group does not decrease if $E(y^{**}) = \frac{m^{2}}{2nN} \geq \frac{n}{2} = E(y^{*})$ or $k \leq \frac{1}{\sqrt{N}}$. Since we found above that it is always true that $E\left(\sum_{i=1}^{N} x_{i}^{*}\right) < E\left(\sum_{i=1}^{N} x_{i}^{**}\right)$
(if $\frac{1}{N} < k < 1$); therefore, a sufficient condition for $RD^* < RD^{**}$ is $k \leq \frac{1}{\sqrt{N}}$.

Moreover, $RD^* < RD^{**}$ if $\frac{n(n + m)}{2m} < \frac{m(nN + m)}{2nN}$ or $k(k^2 + k - 1) < \frac{1}{N}$. Therefore, another sufficient condition for $RD^* < RD^{**}$ is $k^2 + k - 1 \leq 0$ which is equivalent to $k \leq \frac{\sqrt{5} - 1}{2} \approx 0.618$. Combining the above two sufficient conditions we conclude that $RD^* < RD^{**}$ if $\frac{1}{N} < k \leq \max\left(\frac{\sqrt{5} - 1}{2}, \frac{1}{\sqrt{N}}\right)$.

$$\sum_{i=1}^{N} E(U^*_i) = \frac{n^2(N-1)}{2m} < nN - m = \sum_{i=1}^{N} E(U^{**}_i) \quad \text{if} \quad k^2(N-1) - 2kN + 2 < 0 \quad \text{or} \quad k > \overline{k} = \frac{N - \sqrt{N^2 - 2N + 2}}{N - 1}.$$ Since for any $N$, $\overline{k}$ satisfies $\frac{1}{N} < \overline{k} < 1$ then:

1. If $k$ satisfies $\frac{1}{N} < k \leq \overline{k} < 1$, then $\sum_{i=1}^{N} E(U^*_i) \geq \sum_{i=1}^{N} E(U^{**}_i)$.

2. If $k$ satisfies $(\frac{1}{N}) < k < \overline{k}$, then $\sum_{i=1}^{N} E(U^*_i) < \sum_{i=1}^{N} E(U^{**}_i)$.

$$\sum_{i=1}^{N} E(U^*_i) + E(U^*_d) = \frac{n^2(N-1)}{2m} + m - n < nN - m = \sum_{i=1}^{N} E(U^{**}_i) + E(U^{**}_d) \quad \text{if} \quad k^2(N-1) - 2k(N + 1) + 4 < 0 \quad \text{or} \quad k > \overline{k} = \frac{N + 1 - \sqrt{N^2 - 2N + 5}}{N - 1}. \quad \text{Since for any } N, \quad \overline{k}$$ satisfies $\frac{1}{N} < \overline{k} < 1$ then:

1. If $k$ satisfies $\frac{1}{N} < k \leq \overline{k} < 1$, then $\sum_{i=1}^{N} E(U^*_i) + E(U^*_d) \geq \sum_{i=1}^{N} E(U^{**}_i) + E(U^{**}_d)$.

2. If $k$ satisfies $(\frac{1}{N}) < k < \overline{k}$, then $\sum_{i=1}^{N} E(U^*_i) + E(U^*_d) < \sum_{i=1}^{N} E(U^{**}_i) + E(U^{**}_d)$.

**Proposition 3:**

**A. Rent dissipation**

Under an all-pay auction, as a result of the challenging group becoming organized, the rent dissipation of the contest will:

1. decrease if the stake of a member of the challenging group is at least as large as that of the defending group ($k \geq 1$).
2. increase if the stake of a member of the challenging group is small enough

\[ k \leq \text{Max} \left( \frac{\sqrt{3} - 1}{2}, \frac{1}{\sqrt{N}} \right). \]

**B. Total expected payoffs**

Once challenging group has become organized, the total expected payoffs will increase if and only if \( k > k \), where

\[ 1 < k = \frac{N + 1 - \sqrt{N^2 - 2N + 5}}{N - 1} < 1. \]

**C. Expected payoffs of each group**

As the challenging group becomes organized:

1. the expected payoffs of the defending group will not increase.
2. the expected payoffs of the challenging group will increase if and only if

\[ k > \bar{k}, \quad \frac{1}{N} < \bar{k} = \frac{N - \sqrt{N^2 - 2N + 2}}{N - 1} < 1. \]

For proof of proposition 3 see Appendix 2.

Let us explain the economic intuition behind the results of Proposition 3 part A. When the challenging group is not organized, the contest is between one player in the challenging group, and the player in the defending group (the free ride is complete). When there is a central planner in the challenging group, we can treat the challenging group as one player with a (total) value of \( nN \). Therefore, in both cases, the contest is between one player in the challenging group, and the (one) another player in the defending group. Therefore, we have to consider only the relationship between the valuations of two players: When the challenging group is not organized \( n \) and \( m \), and when it is organized \( nN \) and \( m \).

Notice that when the valuations of the two players become more identical the contest is more “symmetric”. Therefore:

1. If \( n \geq m \ (k \geq 1 \text{ - case 1 in 2.2.1}) \) then \( nN > m \ (k > \frac{1}{N} \text{ - case 1 and 2.2.2}) \). In these cases when the challenging group becomes organized, the value of the (player who represents the) challenging group increases to \( nN \). Therefore, the contest is less “symmetric” resulting in a reduction of the expenditure of the defending group; therefore, the total investment in the contest will decrease.
2. If \( nN \leq m \ (k \leq \frac{1}{N} \) - case 2 in 2.2.2) then \( n < m \ (k < 1 \) - case 2 in 2.2.1). In these cases when the challenging group becomes organized, the value of the two players become more identical, and the contest is more “symmetric” resulting in a rise of the expenditure of both groups; therefore, the total investment in the contest will increase.

3. If \( n < m < nN \ (\frac{1}{N} < k < 1 \) - case 2 in 2.2.1 and case 1 in 2.2.2 ), and the challenging group becomes organized, then the value of the two players becomes more/less identical and the contest is more/less “symmetric”. As a result, the expenditure of the challenging group increases, while the effect on the expenditure of the defending group is ambiguous. Let us now explain in which cases the total investment decreases, and in which it increases when the challenging group becomes organized. When the challenging group is not organized, it has a lower value \( n < m \) and the "symmetry" of the contest depends on the "gap" between the two values, \( n \) and \( m \): as long as \( n < m \), given \( m (n) \) the higher (lower) is \( n (m) \) the contest is more “symmetric”. As a result, the expenditure of both group increases. Now let us see what would happen to the "symmetry" of the contest when the challenging group becomes organized in comparison to the first contest. In this case, the challenging group has a higher value \( m < nN \) and as the "symmetry" of the contest decreases, the "gap" becomes between the two values is widened (\( m \) and \( nN \)). For low values of \( N \) the "gap" between the two \( (m \) and \( nN \)) is low, and therefore the contest could be more “symmetric” compared to the first one resulting in a rise of the total investment. If, on the other hand, \( N \) is high, the contest is less “symmetric” in comparison to the first one, which results in a reduction of the total investment.

We can see from proposition 3 parts A2 and B that when
\[
\left( \frac{1}{N} < k < k \leq \text{Max} \left( \frac{\sqrt{5} - 1}{2}, \frac{1}{\sqrt{N}} \right) < 1 \right)
\]
(for example, this would be satisfied if \( k = 0.25 \) and \( N = 10 \)) and the challenging group becomes organized (the challenging group moves from being an underdog to being a favorite), the rent dissipation of the contest increases. However, at the same time the challenging group’s expected payoff
increases,\(^4\) and the expected payoff of the defending group decreases in such a way that the total expected payoffs increase. Thus, the transformation from an underdog to a favorite in the contests has an important impact on the expected payoffs of the contestants and the rent dissipation.

As we have mentioned in the discussion on the Logit contest success function this coincides with the findings of Hurley (1998) who stressed that total expected payoffs would be a better indicator for welfare effects.

3. Concluding remarks

Government intervention often gives rise to contests in which the possible prizes are determined by the status-quo and some new public-policy proposal. Since a proposed policy reform has different implications for different interest groups, these groups make every effort to affect the probability of the approval of the proposed public policy, in their favor. What determines the contestants’ efforts to the proposed policy reform, and, in turn, the change in their probability of winning the contest, are the stakes and the structure of the interest groups.

We consider two types of contest success function: 1. The generalized logit function; 2. The All Pay Auction. We have shown that, if an interest group is governed by a central planner, the rent dissipation of the contest may decrease, and the total expected payoffs increase. On the other hand, it may well be that both the rent dissipation and total expected payoffs increase. In general, we see that the answer to whether the rent dissipation increases or decreases, and its affect on total expected payoffs depend in some sense on who is considered the underdog and who is the favorite. The results we obtained under the all pay auction and the generalized logits contest success functions are not always identical. More specifically, when the stake ration between the two groups \((k)\) is larger or equal to one, then under the generalized logit function the rent dissipation increases, while under the all pay auction the rent dissipation decreases.

In the case of the monopoly story, it is a well-known fact that a regulated monopoly is welfare enhancing, and thus the consumers who wish to regulate the monopoly, also wish to increase social welfare. We have shown that if the ratio of the

\(^4\) It is always true that \(\bar{k} < \bar{\bar{k}}\); therefore, if \(\bar{k} < k\), then \(\bar{\bar{k}} < k\), so the expected payoffs of the challenging group will increase.
stakes between the consumer and the firm are sufficiently high, the consumers will benefit from being governed by a central planner. However, if this ration is not sufficiently high, we may obtain contradicting results for both types of contest success functions.

Even though the probability of the consumers' winning and their expected payoff increases, there may well be an extra benefit: a reduction in the wasteful resources invested in the contest. This last result is not trivial. In such a situation, a government that wishes to decrease wasteful resources should encourage the formation of a consumers group governed by a central planner. This will not only increase the probability of winning, but will also decrease the wasteful recourse.

On the other hand, we have shown that even in the case where the rent dissipation increases, we may find that the total expected payoffs will also increase. This result emphasizes the claim by Hurley (1998), which states that rent dissipation can be a misleading measure of the welfare implications of a contest, when players have asymmetric valuations. If, as commonly assumed in the recent political economic literature, Persson and Tabellini (2000), Grossman and Helpman (2001) and Epstein and Nitzan (2007), the government’s objective function is a weighted average of the expected social welfare and lobbying efforts (rent dissipation), the government would benefit from the challenge of becoming organized since both the expected payoffs and the rent dissipation increase.
References


Appendix 1 - proof of proposition 2

Part A1

We will show that for $k \geq 1$, $RD^* \geq RD^*$ or in other words $G < 0$:

$$G = k^{1+2\alpha} N^{2+\alpha} (1 - N^{1-\alpha}) + 2k^{1+\alpha} N^{1+\alpha} (1 - N) + kN^\alpha (1 - N^{1+\alpha}) + N^\alpha (1 - N^{1+\alpha})$$

(a1)

(a1) can be rewritten

$$G = k^{1+2\alpha} N^{2+\alpha} (1 - N^{1-\alpha}) + kN^\alpha (1 - N^{1+\alpha}) + N^\alpha (1 - N^{1+\alpha})$$

$$+ k^{1+\alpha} N^{1+\alpha} (2 - N) + k^{2\alpha} N^{2+\alpha} (1 - k^{1-\alpha}) - k^{2\alpha} N^2$$

(a2)

Since $N \geq 2$, thus for $k \geq 1 (n \geq m)$ it holds that $G < 0$. This result is independent of $N$.

Part A2

When will $G > 0$ ($RD^* < RD^*$)? For this to hold, $k$ must be sufficiently small and $N$ has to be sufficiently large: for small values of $k$ the values of the first three expressions in $G$ are small (as they are multiplied by $k$):

$$G = k\left[k^\alpha\right]^2 N^{2+\alpha} (1 - N^{1-\alpha}) + 2k^\alpha N^{1+\alpha} (1 - N) + N^\alpha (1 - N^{1+\alpha})$$

$$+ \left[k^\alpha\right]^2 N^2 - N^\alpha \left[N^\alpha - 1\right]$$

(a3)

and if $N$ is sufficiently large the last expression in $G$

$$\left[k^\alpha\right]^2 N^2 - N^\alpha \left[N^\alpha - 1\right]$$

(a4)

may dominate.

For example if $k = 0.003$, $\alpha = 0.7$ and $N = 20,000$ then $G > 0$.

Part B1

$$\sum E(U^*_i) + E(U^*_d) < \sum E(U^*_i) + E(U^*_d), \text{ if:}$$

$$\frac{N^{1-\alpha} n^{1+\alpha} \left[N^{2-\alpha} n^\alpha + m^\alpha (N - \alpha)\right] + m^{1+\alpha} \left[m^\alpha + N^{1-\alpha} n^\alpha (1 - \alpha)\right]}{\left(N^{1-\alpha} n^\alpha + m^\alpha\right)^2} <$$

$$\frac{N^2 n^{1+\alpha} \left[Nn^\alpha + m^\alpha (1 - \alpha)\right] + m^{1+\alpha} \left[m^\alpha + Nn^\alpha (1 - \alpha)\right]}{\left(Nn^\alpha + m^\alpha\right)^2}$$
Substituting $k = \frac{n}{m}$, the condition becomes:

$$
\frac{N^{3-2a}k^{2a}n + N^{2-a}nk^a - \alpha N^{1-a}nk^a + m + mN^{1-a}k^a(1 - \alpha)}{(N^{1-a}k^a + 1)^2} < \frac{N^3nk^{2a} + N^2nk^a(1 - \alpha) + m + mNk^a(1 - \alpha)}{(Nk^a + 1)^3}
$$

or after rearranging:

$$
\alpha N^2k^{1+2a}(N^{1-a} - 1) + 2\alpha Nk^{1+2a}(N - 1) + \alpha k(N^{1+a} - 1) < N^{2-a}k^{2a}(kN - 1 + \alpha) + kN - 1 - \alpha(N^a - 1) + N^{1-a}k^a(kN - 1)(N^{2a} - 1)
$$

(a5)

dividing both sides of (a5) by $k^{1+2a}$ we get:

$$
\alpha N^2(N^{1-a} - 1) + \frac{2\alpha N(N - 1)}{k^a} + \frac{\alpha(N^{1+a} - 1)}{k^{2a}} < N^{2-a}
\left(N - 1 + \frac{\alpha}{k}\right)(N^a - 1) + \left(\frac{N}{k^a} - 1\right) - \frac{\alpha}{k^{2a}}\left(N^a - 1\right)

+ \frac{N^{1-a}}{k^a}
\left(N - 1\right)(N^{2a} - 1)
$$

If $k$ is high enough ($k \to \infty$) we get that the last inequality become:

$$
\alpha N^2(N^{1-a} - 1) < N^{3-a}(N^a - 1)
$$

or:

$$
W = N - (1 + \alpha)N^{1-a} + \alpha > 0
$$

(a6)

We now show that this inequality is satisfied. First $W(\alpha = 0) = 0$ and

$$
\frac{dW}{d\alpha} = 1 + N^{1-a}[(1 + \alpha)\ln N - 1].\quad \text{For } N \geq 3, \quad \frac{dW}{d\alpha} > 0. \quad \text{For } N = 2
$$

$$
\frac{dW}{d\alpha} = 1 + 2^{1-a}[(1 + \alpha)\ln 2 - 1], \quad \text{therefore if } (1 + \alpha)\ln 2 \geq 1 \text{ then } \frac{dW}{d\alpha} > 0 \quad \text{and if}
$$

$$
(1 + \alpha)\ln 2 < 1 \quad \text{then } \frac{dW}{d\alpha} = 1 + 2^{1-a}[(1 + \alpha)\ln 2 - 1] > 1 + 2[\ln 2 - 1] > 0. \quad \text{Since for any } N
$$

and $\alpha$ we found that $\frac{dW}{d\alpha} > 0$ and $W(\alpha = 0) = 0$, therefore $W > 0$. We can conclude that for high values of $k$ we obtain

$$
\sum E(U^*_i) + E(U^*_d) < \sum E(U^*_i) + E(U^*_d)
$$

Part B2

$$
\sum E(U^*_i) + E(U^*_d) > \sum E(U^*_i) + E(U^*_d), \quad \text{if:}
$$
\[
\alpha N^2 k^{1+2\alpha} (N^{1-\alpha} - 1) + 2\alpha Nk^{1+\alpha} (N - 1) + \alpha k (N^{1+\alpha} - 1) > \left[ N^{2-a} k^{2\alpha} (kN - 1 + \alpha) + kN - 1 - \alpha \right] (N^{\alpha} - 1) + N^{1-\alpha} k^a (kN - 1)(N^{2\alpha} - 1) \quad (a7)
\]

The LHS of (a7) is always positive while the RHS is not positive when \( kN - 1 + \alpha \leq 0 \) or \( k \leq \frac{1 - \alpha}{N} \).

**Part C1: Expected payoffs of the defending group**

As a result of the challenging group becoming organized the expected payoffs of the defending group change from

\[
E(U_d^*) = m^{1+\alpha} \left[ m^\alpha + N^{1-\alpha} N^\alpha (1 - \alpha) \right] \frac{(N^{1-\alpha} N^\alpha + m^\alpha)^2}{(N^\alpha + m^\alpha)^2}
\]

to

\[
E(U_d^{**}) = m^{1+\alpha} \left[ m^\alpha + N^{1-\alpha} N^\alpha (1 - \alpha) \right] \frac{(N^\alpha + m^\alpha)^2}{(N^\alpha + m^\alpha)^2}
\]

Since \( \frac{\partial E(U_d^{**})}{\partial N} < 0 \) and moving from \( E(U_d^*) \) to \( E(U_d^{**}) \), \( N^{1-\alpha} \) increases to \( N \) therefore \( E(U_d^*) > E(U_d^{**}) \).

**Part C2: Expected payoffs of the challenging group**

\[
\sum E(U_i^*) > \sum E(U_i^{**}) \text{ if:}
\]

\[
\frac{N^3 k^{2\alpha} - N^{2-a} nk^\alpha - \alpha N^{1-\alpha} nk^\alpha}{N^2 k^{2\alpha} + 2N^{1-\alpha} k^\alpha + 1} > \frac{N^3 nk^{2\alpha} + N^2 nk^\alpha (1 - \alpha)}{N^2 k^{2\alpha} + 2Nk^\alpha + 1}
\]

or after rearranging:

\[
\alpha Nk^\alpha (N - 2) + \alpha (N^{1+\alpha} - 1) Nk^2 a \left( N^{1-\alpha} - 1 \right) > N \left( - k^{2\alpha} + 1 \right) (N^\alpha - 1) + N^2 k^a \left( N^{2\alpha} - 1 \right) \quad (a8)
\]

Thus for low values of \( k \) (approaching zero) we obtain that (a8) is equivalent to \( \alpha (N^{1+\alpha} - 1) > N^{1+\alpha} - N \). We can conclude that for low values of \( k \) and \( \alpha \rightarrow 1 \) we obtain \( \sum E(U_i^*) > \sum E(U_i^{**}) \).

\[
\sum E(U_i^*) < \sum E(U_i^{**}) \text{ if:}
\]

\[
\alpha Nk^\alpha (N - 2) + \alpha (N^{1+\alpha} - 1) Nk^2 a \left( N^{1-\alpha} - 1 \right) < N \left( - k^{2\alpha} + 1 \right) (N^\alpha - 1) + N^2 k^a \left( N^{2\alpha} - 1 \right) \quad (a9)
\]
dividing both sides of (a9) by $k^{2\alpha}$ we get:

$$\frac{\alpha N(N-2)}{k^{\alpha}} + \alpha \left(\frac{N^{1+\alpha} - 1}{k^{2\alpha}}\right) \alpha N^{2} \left(N^{1-\alpha} - 1\right) < N^{2-\alpha} \left(\frac{1}{k^{2\alpha}}\right) \left(N^{-\alpha} - 1\right) + \frac{N^{2-\alpha} \left(N^{2\alpha} - 1\right)}{k^{\alpha}}$$

If $k$ is high enough ($k \to \infty$) we get that the last inequality become: $\alpha N^{2} \left(N^{1-\alpha} - 1\right) < N^{3-\alpha} \left(N^{-\alpha} - 1\right)$ or $N - (1 + \alpha) N^{1-\alpha} + \alpha > 0$. This inequality is identical to inequality (a6). Since for high values of $k$ ($k$ approaching infinity), we proved in part B1 that inequality (a6) is always satisfied therefore for high values of $k$ we obtain $\sum E(U_i^1) < \sum E(U_i^{*1})$.

Appendix 2 - proof of proposition 3

Part A1

This result is concluded with 2.2.3 case 1.

Part A2

If $k \leq \frac{1}{N}$ we found in 2.2.3 case 1 that $RD^* < RD^{**}$. If $\frac{1}{N} < k < 1$ we found in 2.2.3 case 3 that a sufficient conditions for $RD^* < RD^{**}$ is $\frac{1}{N} < k \leq \text{Max} \left(\frac{\sqrt{5} - 1}{2}, \frac{1}{\sqrt{N}}\right)$. Therefore $RD^* < RD^{**}$ if $k \leq \text{Max} \left(\frac{\sqrt{5} - 1}{2}, \frac{1}{\sqrt{N}}\right)$.

Part B

If $k \geq 1$ we found in 2.2.3 case 1 that $\sum E(U_i^1) + E(U_i^{*1}) < \sum E(U_i^{*1}) + E(U_i^{**})$. If $k \leq \frac{1}{N}$ we found in 2.2.3 case 2 that $\sum E(U_i^1) + E(U_i^{*1}) > \sum E(U_i^{*1}) + E(U_i^{**})$. If $\frac{1}{N} < k < 1$ we found in 2.2.3 case 3 that $\sum E(U_i^1) + E(U_i^{*1}) < \sum E(U_i^{*1}) + E(U_i^{**})$ if
and only if \( k \) satisfies \( \frac{1}{N} < k < 1 \), where \( \frac{1}{N} < k = \frac{N + 1 - \sqrt{N^2 - 2N + 5}}{N - 1} < 1 \).

Therefore \( \sum E(U_i^+) + E(U_d^+) < \sum E(U_i^{**}) + E(U_d^{**}) \) if and only if \( k > \bar{k} \).

**Part C1: Expected payoffs of the defending group**

If \( k \geq 1 \) we found in 2.2.3 case 1 that \( E(U_d^+) = E(U_d^{**}) = 0 \). If \( k < 1 \) we found in 2.2.3 cases 2 and 3 that \( E(U_d^+) > E(U_d^{**}) \). Therefore in any case the expected payoffs of the defending group will not increase.

**Part C2: Expected payoffs of the challenging group**

If \( k \geq 1 \) we found in 2.2.3 case 1 that \( \sum_{i=1}^{N} E(U_i^+) < \sum_{i=1}^{N} E(U_i^{**}) \). If \( \frac{1}{N} \leq k \leq 1 \) we found in 2.2.3 case 2 that \( \sum_{i=1}^{N} E(U_i^+) > \sum_{i=1}^{N} E(U_i^{**}) \). If \( \frac{1}{N} < k < 1 \) we found in 2.2.3 case 3 that \( \sum_{i=1}^{N} E(U_i^+) < \sum_{i=1}^{N} E(U_i^{**}) \) if and only if \( k \) satisfies \( \frac{1}{N} < \bar{k} < k < 1 \), where

\[
\frac{1}{N} < \bar{k} = \frac{N - \sqrt{N^2 - 2N + 2}}{N - 1} < 1. 
\]

Therefore \( \sum_{i=1}^{N} E(U_i^+) < \sum_{i=1}^{N} E(U_i^{**}) \) if and only if \( k > \bar{k} \).
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