Consumer Absenteeism, Search, Advertising, and Sticky Prices

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Abstract

This paper shows that prices may be sticky when buyers must search to determine the current market price and there is uncertainty about the expected duration of cost changes. Specifically, during periods when costs, and hence prices are high, low valuation consumers optimally stop searching and consequently are uninformed about price changes. Then, when costs go down, sellers must advertise to inform those consumers about price cuts. If advertising is costly, relative to single period profit, advertising is profitable only if the cost cut is likely to persist, but not if it is likely to be short lived. Thus, if sellers are initially uncertain about the expected longevity of a cost cut, they might adopt a 'watch and wait' strategy, delaying price reductions until better information becomes available. Importantly, it is shown that the same logic does not apply to cost increases. Thus the model is consistent with asymmetric price rigidity (e.g., Peltzman (2000)).

Keywords: search, advertising, asymmetric price adjustment, sticky prices, absentee consumers

1. Introduction

There is a consensus that in many contexts prices change less frequently than costs - they are sticky. It has been argued that prices are sticky if there are real costs (menu costs) of changing prices (e.g., Sheshinski and Weiss, 1977), if consumers are antagonized by frequent prices changes (Rotemberg, 2005), if it is costly to process information about cost changes (Mankiw and Reis (2002),
Reis (2006)), if consumers are inattentive to small changes (Levy et al (2008)), if consumers form habits in individual goods which lock them in with specific sellers (Nakamura and Steinsson (2005)), to deter consumers from searching for lower prices (Cabral and Fishman (2010)) or in a monetary model with search frictions (Head, Liu Menzio and Wright (2010)).

This paper shows that prices may be sticky when buyers must incur search costs to determine the current market price and there is uncertainty about the expected duration of cost changes. Specifically, during periods when costs, and hence prices are high, low valuation consumers optimally stop searching and consequently are uninformed about price changes. In that case, when costs go down, sellers must advertise to inform those consumers about price cuts. If advertising is sufficiently costly, relative to single period profit, this may only be profitable if the cost reduction is likely to persist, but not if it is likely to be short lived. Thus, if sellers are initially uncertain about the expected longevity of a cost cut, it might be optimal to adopt a ‘watch and wait’ strategy, delaying price reductions until better information becomes available. Short term price rigidity is the result.

Importantly, the same logic does not necessarily apply to cost increases. Specifically, if prices have been low, and low valuation consumers have been searching, they are already ‘in the market’ at the time the price increases. If consumers recognize that the price increase might be transient, they may optimally continue to search until it becomes clear that the high price is likely to persist. In that case, there is no reason for sellers to delay price increases. Such asymmetric price dynamics are consistent with the empirical findings of Peltzman (2000) and others.

2. Model

Time is discrete and the horizon is infinite. A monopoly seller sells a homogenous product to a continuum of consumers. Consumers are infinitely lived and are of two types, low (\( l \)) valuation and high (\( h \)) valuation. The former get utility \( u_l \) from a consuming a unit and the latter get utility \( u_h \) from a unit, \( u_h > u_l \). A consumer demands one unit (at most) at each period and storage of the good is not possible. The proportion of high valuation consumers is \( \alpha < 1 \). The discount factor is denoted \( \delta < 1 \).

\(^1\)Cabral and Fishman (2010), Lewis(2005), Tappata (2006) and Yang and Ye (2006)) develop search theoretic models which lead to asymmetric price adjustment.
The production cost at period $t$ is denoted $c_t$. At any period, $c_t$ may assume one of two possible values: $c_L$ (the low cost) or $c_H > c_L$ (the high cost). (Indexes with capital $L$ and $H$ refer to cost states, with small $l$ and $h$ refer to consumer types (valuations)).

The idea that changes in production costs may be of either short or long duration is formulated as follows. There are two cost states, the low cost state $L$ and the high cost state $H$. I denote the state at period $t$ as $s_t$, $s_t = i, i \in \{L, H\}$. In state $L$, $c_t = c_L$ "most of the time" and in state $H$, $c_t = c_H$ "most of the time". Specifically, at each period in which the cost state is $L$, $c_t = c_L$ with probability $\eta$, $1 > \eta > 0.5$. Similarly, at each period in which the cost state is $H$, $c_t = c_H$ with probability $\eta$. The cost state itself evolves as a Markov chain; If the state is $L(H)$ at period $t$, then at the following period the state is $L(H)$ with probability $\gamma$ and $H(L)$ with probability $1 - \gamma$, where $\gamma > 0.5$.

We think of the 'state' as representing long run 'market fundamentals', such as general economy wide level of costs, which change only infrequently, while the probability $1 - \eta$, with which the cost diverges from the underlying state (i.e., the probability that $c_t = c_j$ when $s_t = i, j \neq i$) captures occasional transitory shocks which cause the cost to temporarily diverge from its longer run level (consistent with the state), such as a temporary firm-specific shock to costs. The realizations of $c_t$ at successive periods, $t$ and $t+1$, such that $s_t = s_{t+1}$, are assumed to be i.i.d.

At every period $t$ the seller learns the current cost, $c_t$, but only learns the cost state which was in effect at the preceding period, $s_{t-1}$; the current cost state, $s_t$, is only learned with a one period lag, at the following period.

2.1. Consumers’ Information

Consumers are uninformed about production costs, past or present. Consumers are also imperfectly informed about prices. Specifically, unless the current price is advertised (as described below), a consumer must incur a search cost $\phi > 0$ to learn the current price (for example, by going to the store or consulting its website). In contrast to conventional search models in which the search cost is the buyers’ cost of comparing prices of different sellers, here it is the cost of searching over time at the same firm.

The utility of a consumer with valuation $u$ who searches and buys a unit at the price is $p$ is $u - p - \phi$. Thus a consumer with valuation $u$ will only search if $Ep \leq u - \phi$, where $Ep$ is the expected price that period. We assume that

\footnote{Assume that at the first period, $p_1$ is costlessly observed by all consumers.}
consumers remember the prices they have observed in the past and may form expectations about the current price on the basis of those prices.

At the beginning of any period, the seller can, at a (fixed) advertising cost $A$, advertise the current price. Advertising commits the seller to the advertised price at that period. For simplicity I assume that an advertised price is communicated to all consumers before they enter the market. Thus consumers have no need to search when the price is advertised.

The following relationship between consumer demand and production costs is assumed:

$$u_h > c_H > u_l$$

$$\alpha(u_h - c_l) < u_l - c_l$$

The preceding assumptions imply that if $\phi = 0$, then the unique equilibrium prices are $p_t = u_h$ when $c_t = c_H$ and $p_t = u_l$ when $c_t = c_l$. Similarly, if $\phi > 0$ but $A$ is sufficiently small, the unique equilibrium prices are $p_t = u_h - \phi$ when the $c_t = c_H$ and $p_t = u_l - \phi$ when $c_t = c_l$. In both these benchmark cases, $L$ consumers buy whenever the cost is $c_L$ and only $H$ consumers buy when the cost is $c_H$.

Things are more complex if $\phi > 0$ and $A$ is large relative to one period profits. This is because then consumers’ decision whether or not to search depends on what prices they expect, and these expectations may be quite arbitrary. In particular, if $\delta$ is sufficiently large, a plethora of equilibria can be constructed for this infinite horizon game if $\delta$ is sufficiently near 1 - for example, equilibria in which prices depend on calendar time and/or ‘irrelevant’ history. Characterization of

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4Suppose it costs $a$ for an ad to reach an individual consumer - then if its optimal to advertise for one consumer, its optimal for any number, so we can think of $A$ as an, where $n$ is the number of customers the seller is targeting.

5I assume that the seller cannot commit to prices at future periods. If it could, price rigidity could result simply from the fact that the seller can’t change its price while committed to a fixed price.

6More precisely this is the case if $A = 0$. If $A > 0$ is sufficiently small, the price must be sufficiently near $u_l$ at low cost periods and near $u_h$ at high cost periods; otherwise the seller would optimally advertise those prices.
equilibria thus requires that some structure be imposed by restricting attention to 'reasonable' equilibria. An intuitively appealing class of equilibria is the following in which the seller and buyers have simple and stationary strategies.

**Simple equilibria**: These equilibria are characterized by prices $p_l$ and $p_h$ such that (i) the price is $p_l$ at every period in which all consumers buy and the price is $p_h$ at all periods in which only $h$ consumers buy. (ii) type $i$ consumers search at period $t$ if $p_{t-1} \leq p_i$.

The idea behind (ii) in the preceding definition is that $l$ consumers will continue to search if they were charged an 'acceptable' (i.e., which provides them with sufficient surplus) price when they last purchased. Thus, if its price is low enough, the seller can keep $l$ customers buying without the need to continuously advertise.

This restriction still allows for a wide range of equilibria. For example, there are always equilibria in which only $H$ consumers buy; For example, let $p_l < c_l$ and let $L$ consumers expect prices $> u_i - \phi$ unless past prices $< c_l$; Then the seller must advertise at every period at which it wants to sell to $L$ consumers, which isn’t optimal if $A$ is large relative to single period profits$^7$.

I shall instead focus on the more interesting stationary equilibria in which, in line with the benchmark cases discussed above, only $h$ consumers buy when the cost is high but all consumers buy when the cost is low, at least some of the time. Henceforth, the term equilibrium refers to simple equilibria in which $L$ consumers buy when the cost is low.

Equilibria are said to be characterized by downward price rigidity if, generally, the price doesn’t come down when the cost goes down. I shall show that if $A$, $\gamma$ and $\eta$ are sufficiently large (that is, the cost of advertising is high and costs don’t change too frequently), then all stationary equilibria in which $l$ consumers buy during the low cost state are characterized by downward price rigidity.

The reason is straightforward. If costs change infrequently, then once prices turn high (in response to high costs), consumers expect high prices, optimally do not search and stay out of the market. Thus, once the cost comes down, consumers can learn about a price cut only if the seller advertises it. If advertising is costly, relative to single period profits, advertising is only profitable if the cost change is likely to persist, but not if it is likely to be transitory. Thus, if the seller is initially unsure about the likely duration of a cost change, it optimally pursues a watch and wait strategy, delaying a price change until it is more confident of its duration.

$^7$By the same token, there always exists a no-trade equilibrium in which all consumers expect unadvertised prices to be $> u_h$ if $A$ is large.
I shall denote by \( t \in [i, c_i] \) a period \( t \) such that \( s_{t-1} = i, c_i = c_i \) as, \( i = L, H \). (only \( s_{t-1} \) is relevant because \( s_t \) is unobserved) (and similarly by \( t \in [i, c_j] \) a period \( t \) such that \( s_{t-1} = i, c_i = c_j \) as, \( i = L, H, j \neq i \)).

**Proposition 2.1.** Let \( A^* = \frac{u_l - \phi - c_l}{0.5} \). If \( A > A^* \), \( \gamma \) and \( \eta \) are sufficiently large, and \( \phi \) is sufficiently small, then stationary equilibria in which \( l \) consumers buy at periods \( t \in [L, c_L] \) are characterized by downward price rigidity.

**Proof:** Since \( l \) consumers buy when the price is \( p_l \), then \( p_l \leq u_l - \phi \). And since \( h \) consumers buy when the price is \( p_h \), \( p_h \geq c_H \).

I shall show that under the conditions of the proposition, in any stationary-equilibrium, the price is generally \( p_h \) at periods \( t \in [H, c_L] \). Suppose the contrary that the price is \( p_l \) whenever \( c_t = c_L \) - both if \( t \in [H, c_L] \) or if \( t \in [L, c_L] \).

**Step 1.** If \( \gamma \) and \( \eta \) are sufficiently large, then at a period \( t \in [H, c_H] \), \( p_t = p_h \). This is because \( h \) consumers will continue to search and buy as long as the price is \( p_h \). The only possible reason for pricing at \( p_l \), below cost, can be to keep \( l \) consumers searching. Thus if \( \gamma \) and \( \eta \) are sufficiently high, so that the cost is expected to remain high with sufficiently high probability, it is more profitable to sell only to \( h \) consumers at the price \( p_h \), at least until and if the cost goes down again.

**Step 2.** Consider an \( l \) consumer who observes the price \( p_h \) for 2 consecutive periods, \( t \) and \( t+1 \). Then, since we’re assuming that the price is \( p_l \) whenever the cost is low, she must conclude that at those periods the cost was \( c_H \). Then, given that \( c_t = c_{t+1} = c_H \), if \( \gamma \) and \( \eta \rightarrow 1 \), the posterior probability that \( s_{t+1} = H \rightarrow 1 \). Thus if \( \gamma \) and \( \eta \) are sufficiently large, then, by step 1, they should expect the price to be \( p_h \) at the following periods with sufficiently high probability that, since \( p_h > u_l - s \), they optimally do not search after period \( t+1 \) until a price \( \leq u_l - \phi \) is advertised.

**Step 3.** Consider a period \( t' \in [H, c_L] \) preceded by \( t' - 1, \ t' - 2 \in [H, c_H] \). Then by step 1, \( p_{t' - 1} = p_{t' - 2} = p_h \).

Suppose \( l \) consumers didn’t search at period \( t' - 1 \). Then without having received new information there was no reason for them to search at period \( t' \) and thus will stay out of the market until a price \( \leq u_l - \phi \) is advertised.

If \( l \) consumers searched at period \( t' - 1 \), they must have also searched at period \( t' - 2 \); otherwise, they would have no reason to search at \( t' - 1 \) without receiving.

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8This is obviously the case if \( \gamma = \eta = 1 \) and is therefore also the case if \( \gamma \) and \( \eta \) are sufficiently large.
new information (i.e., unless a low price was advertised). In that case, having observed \( p_{t-1} \) and \( p_{t-2} \), then by step 2, \( L \) consumers won’t search at period \( t' \).

Thus in any case \( l \) consumers don’t search at period \( t' \) and thus will only buy that period if a price \( u_l - \phi \) is advertised.

Let \( V^A_{t'} \) be the seller’s expected profit, evaluated at period \( t' \), if it advertises \( p_t \) at period \( t' \), let \( V_{t'+1}(L) \) and \( V_{t'+1}(H) \) be, respectively, the discounted profit evaluated at period \( t' + 1 \) if it turns out that \( s_{t'} = L \), and if \( s_{t'} = H \). Then

\[
V^A_{t'} = -A + p_t - c_t + \delta \left[ \theta V_{t'+1}(L) + (1 - \theta) V_{t'+1}(H) \right]
\]  

(2.3)

where \( \theta \) is the posterior probability that \( s_t = L \) which, by Bayes rule is given by:

\[
\theta = \frac{\alpha(1 - \eta)}{\gamma(1 - \eta) + (1 - \gamma) \eta}
\]  

(2.4)

Consider the alternative strategy of setting \( p_{t'} = p_h \), and advertising at period \( t'+1 \) if and only if it turns out that \( s_{t'} = L \) (this won’t change \( l \) consumers’ behavior after period \( t' \), since (by (ii) of the definition of stationary equilibrium), once \( p_t \) is advertised, they will search as long as the price continues to be \( p_t \)).

Denoting the profit from this strategy as \( V^{NA}_{t'} \), we have:

\[
V^{NA}_{t'} = \alpha(p_h - c_l) + \delta \left[ \theta V_{t'+1}(L) - A + (1 - \theta) V_{t'+1}(H) \right]
\]  

(2.5)

Thus, \( V^A_{t'} < V^{NA}_{t'} \) if \( A < \frac{p_l - c_l - \alpha(p_h - c_l)}{\theta \delta} \). Note that the RHS of the preceding inequality attains its maximal value if \( p_h = c_H \) and \( p_l = u_l - \phi \), and that \( \theta \to 0.5 \) as \( \gamma \) and \( \eta \to 1 \). Thus, if \( A^* \equiv \frac{u_l - \phi - c_l}{0.5 \delta} \) then for sufficiently large \( \gamma, \eta \) and \( \delta \), and sufficiently small \( \phi \), if \( A > A^* \), then \( p_{t'} = p_h \). The same argument implies that the price continues to be \( p_h \) after period \( t' \) as long as \( s_t \) continues to be \( H \).

Thus, once the state is \( H \), and the cost is \( c_H \) for at least two consecutive periods, the price continues to be \( p_h \) whether the cost is high or low until the state changes.

End proof.

Proposition 1 is our main result. It shows that in this model, downward price rigidity is not an artifact of an arbitrary equilibrium construction but an inherent

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\( ^9 \) [As \( \gamma \) and \( \eta \to 1 \), \( V_{t'+1}(L) \to \frac{p_l - c_l}{1 - \delta} \) and \( V_{t'+1}(H) \to \frac{p_h - c_H}{1 - \delta} \), \( \theta \delta \left[ \delta (p_h - c_l) - A \right] + (1 - \theta) \delta (p_h - c_H) ] \)
feature of (stationary) equilibria when advertising costs are large and costs are sufficiently persistent.

Importantly, there is no parallel argument which implies *upward* price rigidity. As was argued above, downward price rigidity is due to the fact that when costs are high, \( L \) consumers don’t search, are out of the market, and hence will be unaware of a price cut unless it is advertised. Therefore, if advertising is too costly if a cost cut is likely to prove transient, prices will be slow to adjust to cost decrease. By contrast, in the case of cost increase in the wake of a stretch of low costs and low prices, \( L \) consumers do search at the time of the price increases and therefore, if the search cost is relatively small, it makes sense for those consumers to reenter the market and search once more in case the price increase turns out to be temporary. In that case, temporarily raising the price won’t necessary lead to additional advertising expenditures. Thus the model does not necessarily imply upward price rigidity.

Based on the preceding analysis, I construct simple, stationary equilibria characterized by asymmetric price rigidity. In these equilibria, the price increases without delay in response to a cost increase "most of the time" but, by contrast, decreases in response to a cost decrease with at least a one period delay "most of the time". Thus prices generally respond more quickly to a cost increase than to a cost decrease.

2.2. Constructing Simple Equilibria

**Proposition:**

If \( A > A^* \), \( \gamma \) and \( \delta \) are sufficiently large, and \( \phi \) is sufficiently small, there exist simple equilibria with \( c_L < p_t < u_t - 2\phi \) and \( p_h \leq u_h - \phi \) such that:

1. (Si) If \( t \in [H, c_H] \), \( p_t = p_h \)
2. (Sii) if \( t \in (L, c_L) \), \( p_t = p_l \); if consumers don’t search that period, advertise \( p_l \)
3. (Siii) If \( t \in [L, c_H] \): (a) \( p_t = p_h \) if \( p_{t-1} = p_l \). (b) \( p_t = p_l \) if \( p_{t-1} = p_h \).
4. (Siv) if \( t \in (H, c_L) \): let \( t' < t \) be such that \( s_{t'} = L, s_{t'} = H \) for all \( \tau > t' \) (i.e., the state was \( L \) prior to period \( t' \) and has been uninterruptedly \( H \) since then). (a) \( p_t = p_h \) if there exist \( t'' > t' \) such that \( t'', t'' + 1 \in (H, c_H) \). (b) Otherwise, \( p_t = p_l \).

These strategies produce the following price/search cycle. Type \( h \) consumers search and buy every period. When the cost state turns \( H \), once the cost is \( c_H \) for at least two consecutive periods, \( l \) consumers stop searching and the price is \( p_h \) until the first period \( \in (L, c_l) \). At that point the price \( p_l \) is advertised. After
that \( l \) consumers search until the price is \( p_h \) for two consecutive periods, at which point they stop searching and the cycle begins again.

**Proof:** All the following arguments apply for \( \delta \) sufficiently close to 1. Statements about consumers' search strategy like "Search at period \( t \) if \( p_{t-1} < \) " or "don't search if \( p_{t-1} \)" mean that those consumers observed \( p_{t-1} \), either because they searched at period \( t \) or the price was advertised that period.

Let consumers' search strategies be as follows.

**H consumers' strategy:** Search at period \( t > 1 \) if and only if \( p_t < p_h \).

**L consumers' strategy:**

Search at period \( t > 1 \) if: (Li) \( p_t \leq p_l \); or: (Lii) \( p_t = p_h \) and \( p_{t-2} \leq p_l \). Otherwise don't search.

**Proof that the seller's pricing strategy is a best response to consumers' strategy:**

(Si) Suppose \( \gamma = \eta = 1 \). Since \( L \) consumers wont buy at a price greater than \( u_l - \phi < c_H \) and \( h \) consumers search if the price is \( \leq p_h \), \( p_t < p_h \) is not optimal. If \( p_t = p_h \), \( h \) consumers will continue to search and buy as long as the price is \( p_h \). If \( u_H > p_t > p_h \), \( h \) consumers might pay a higher price at period \( t \) but wont search again until a lower price is advertised, at the cost of \( A \). Thus, \( p_h \) is optimal if \( A > \alpha(u_h - p_h) \), and are thus is also optimal if \( \gamma, \eta < 1 \) are sufficiently large.

(Sii) Suppose \( \gamma = \eta = 1 \). Since all consumers search if \( p_t = p_l \), \( p_t < p_l \) isn't optimal. If \( p_t = p_l \), all consumers continue to search and buy as long as the price is \( p_l \). If \( u_l \geq p_t > p_l \), \( L \) consumers might pay more at period \( t \) but wont search again until \( p_l \) is advertised and thus \( p_l \) is optimal if \( A > u_l - p_l \). Thus \( p_l \) is also optimal if \( \gamma, \eta < 1 \) are sufficiently large.

(Siii) (a) In this case, (since \( l \) consumers observed \( p_{t-1} \), as the price is \( p_t \) only at periods in which \( l \) consumers buy), \( l \) consumers search strategy is to search at period \( t+1 \) even if \( p_t = p_h \). Thus \( p_h \) increases profit at period \( t \) without affecting future options and thus is optimal.

(Siii) (b) Since \( t \in [L, c_H] \), \( s_{t-1} = L \). Therefore the posterior probability that \( s_t = L \to 0.5 \) as \( \gamma, \eta \to 1 \). In this case, by (Lii), \( l \) consumers will search next period only if \( p_t \leq p_l \). Thus if \( s_t = L \) and \( p_t > p_l \) it will be necessary to advertise to sell to \( L \) consumers after period \( t \) while if \( p_t = p_l \), it will not be necessary to advertise. Thus if \( \gamma, \eta \) and \( A \) are sufficiently large, \( p_t = p_l \) is optimal.

(Siv) (a) In this case, by (Si), \( p_vw = p_{v+1} = p_h \) and thus \( l \) consumers will have stopped searching by period \( t \). Thus \( l \) consumers will only buy from period \( t \) and onwards if the seller advertises. Then the arguments in the proof of proposition 1 show that it is optimal to defer advertising and set \( p_t = p_h \) if \( A \) is sufficiently
(b) In this case \(l\) consumers search at period \(t\). If \(p_t = p_l\), \(L\) consumers will search at period \(t + 1\), whereas if \(p_t > p_l\), they won’t search after period \(t\) until the seller advertises. Since \(t \in [H, cL]\), \(s_{t-1} = L\). Therefore the posterior probability that \(s_t = L \rightarrow 0.5 \text{ as } \gamma, \eta \rightarrow 1\). Thus if \(\gamma, \eta\) and \(A\) are sufficiently large, \(p_t = p_l\) is optimal.

**Proof that the consumers search strategy is an optimal response to the sellers strategy:**

Obviously the \(H\) consumers’ search strategy is optimal.

We must show that the \(L\) consumers’ search strategy is an optimal response to the seller’s pricing strategy.

Suppose \(p_{t-1} = p_l\). Then the sellers price strategy implies that either \(t - 1 \in [L, cL]\) or \(t - 1 \in [H, cL]\). As \(\gamma, \eta \rightarrow 1\), the probability that \(t \in [L, cL] \rightarrow 1\), therefore the probability that \(p_t = p_l \rightarrow 1\), and thus the consumers’ expected utility from searching at period \(t \rightarrow (u_l - \phi - p_l) > \phi > 0 \text{ (substituting } p_l < u_l - 2\phi)\). Thus for sufficiently large \(\gamma, \eta\), \(l\) consumers get positive expected utility from searching at period \(t + 1\).

Suppose \(p_{t-1} = p_h\) and \(p_{t-2} = p_l\). Then, the sellers price strategy implies that either \(t - 2 \in [L, cL]\) or \(t - 2 \in [H, cL]\). The fact that \(p_{t-1} = p_h\) implies that \(c_{t-1} = c_h\). As \(\gamma, \eta \rightarrow 1\), the probability that \(t - 2 \in [L, cL] \rightarrow 1\) and thus, given that \(c_{t-1} = c_h\), the consumers’ posterior probability that \(s_t = H \rightarrow \theta = \ldots \rightarrow 0.5 \text{ as } \gamma, \eta \rightarrow 1\). Thus the probability that \(p_{t+1} = p_l \rightarrow 0.5\) and thus the consumers expected utility from searching at period \(t + 1 \rightarrow 0.5(u_l - \phi - p_l) > 0 \text{ (substituting } p_l < u_l - 2\phi)\). Thus \(l\) consumers get positive expected utility from searching at period \(t\) if \(\gamma\) and \(\eta\) are sufficiently large.

Suppose \(p_{t-1} = p_{t-2} = p_h\). Then \(c_{t-1} = c_{t-1} = c_h\). Then as \(\gamma, \eta \rightarrow 1\), the probability that \(s_t = H\) and that therefore \(p_t = p_h \rightarrow 1\) and thus the consumer’s expected utility from searching at period \(t \rightarrow -\phi < 0\). Thus for sufficiently large \(\gamma, \eta\) it is optimal for \(L\) consumers not to search at period \(t\).

Finally, if an \(L\) consumer did not observe \(p_{t-1}\), then, it was optimal for her not to search at period \(t - 1\), and therefore, if \(\gamma\) and \(\eta\) are sufficiently large, unless a price is advertised at period \(t\) she has no reason to expect positive utility from searching at period \(t\) and thus optimally doesn’t search. This completes the proof.

End proof.

The equilibrium constructed above has the feature that prices are downwardly sticky but upwardly flexible. Specifically, if since the state last turned \(H\), the cost
has been $c_H$ for at least two consecutive periods, then, when the cost comes down, the price does not change immediately. If it turns out that the state has changed to $L$, then the price comes down with a one period delay. If it turns out that the state hasn’t changed, the price continues to be high. By contrast, when the cost goes up, the price increases without delay. A price increase is only delayed if the cost goes up for a second consecutive period when the state stays $L$. Thus, ‘most of the time’ prices increase more quickly in response to cost increases than they decrease in response to cost decreases. As established by proposition 1, this is not an artifact of a contrived equilibrium construction but arises naturally in the context of the model.

3. Model without Advertising

We may formulate an alternative version of the model without advertising. In contrast to the base model, in which it was assumed that consumers are only informed about past prices which they observed while they are in the market, suppose instead that at any period $t$ a consumer costlessly learns past prices $p_{t-1}, p_{t-2}, \ldots$ whether or not she searched at those periods. But, as in the base model, she only learns the current price, $p_t$, if she searches (there is now no advertising). Now very similar arguments to those in the proof of proposition 1 establish that if $\gamma$ is sufficiently large, equilibria in which $L$ consumers buy at low cost periods must be characterized by short term price rigidity. Again, the reason is that if costs change infrequently, $L$ consumers stop searching once costs and prices have been high for some time. Thus, when the price goes down, at period $t$, $L$ consumers will only be aware of this at period $t + 1$. In that case, reducing the cost at period $t$ lowers profit at period $t$ and only increases profit after period $t$ if it turns out the state has turned $L$. Hence a price decrease is delayed until it becomes clear whether the cost reduction is likely to persist. Again, parallel argument for upward rigidity do not apply because if $l$ consumers observe a price increase, they may still opt to keep searching if the cost change is likely to be temporary. The details of the arguments are very similar to those of the preceding analysis and are omitted.
4. Bibliography


