Brain Drain and Development Traps*

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Abstract

This paper links the two fields of “development traps” and “brain drain”. We construct a model which integrates endogenous international migration into a simple growth model. As a result the dynamics of the economy can feature some underdevelopment traps: an economy starting with a low level of human capital can be caught in a vicious circle where low level of human capital leads to low wages, and low wages leads to emigration of valuable human capital. We also show that our model displays a rich array of different dynamic regimes, including the above traps, but other regimes as well, and we link explicitly the nature of the regimes to technology and policy parameters.

Keywords: brain drain; development traps; human capital; migration.

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1 Introduction

During the last decade, the improvement in modeling tools has permitted to tackle the existence of multiple equilibria in macroeconomics. More specifically, the multiple equilibria apparatus has been applied quite intensively to the explanation of development traps, also coined poverty traps, and various mechanisms have been developed which allow to analyze how an economy can be characterized by multiple equilibria and find itself historically trapped in an inferior equilibrium.

The diverse mechanisms developed have focused notably on market size, financial development, low investment, learning by doing externalities, demographic aspects or contagious social norms. Recent synthetic accounts of the literature on this very rich area can be found in Azariadis and Stachurski (2005), Bowles, Durlauf and Hoff (2006) and Matsuyama (2008).

The purpose of this paper is to propose another mechanism for a poverty trap, through the endogenous migration of the elites. In order to do so, we link the two fields of “development traps” and “brain drain”.\textsuperscript{1} We construct a model which integrates endogenous international migration into a simple growth model, and we show that the existence of brain drain can lead to vicious circles and to poverty traps.

In our model the possibility of a vicious circle is due to the combination of two elements. The first is that, as in Romer (1990), there are positive externalities between different lines of production using skilled workers. These positive externalities will result in productivity and real wages being possibly increasing with the skilled population. The second element is the migration mechanism: If the real wage at home is lower than the real wage abroad, then a part of the population will emigrate.

These two elements, when put together, can lead to vicious circles in the following way. If the high skilled population is low to start with, the resulting wage is low because of the above externalities, and therefore many workers emigrate abroad. This in turn reduces productivity and the real wage further, which will lead to further migration and so on. We have clearly a vicious circle, which can create an underdevelopment trap, because of the loss of skilled workers who had accumulated valuable human capital.

If in the contrary the economy starts with a high level of skilled population and therefore productivity, most workers will choose to stay in the home

\textsuperscript{1}The flight of the elites with high human capital has given rise to a large literature on “brain drain”, which stresses that an important mechanism of impoverishment for developing countries is the flight of skilled elites towards countries with higher standards of living. A recent overview of that issue is Docquier and Rappoport (2008). Further references to the literature are found in section 2 below.
country, which will itself lead to high productivity and low emigration.

This line of reasoning has close ties to De la Croix and Docquier (2010), which actually motivated this paper. They combine a migration function based on wage differentials with a production function exhibiting Lucas (1988) type externalities. Their dynamics is stochastic, being led by “sunspots”, through which the economy somehow “alternates” between two types of trajectories. The focus of their paper is thus on coordination failures.

Although it has similar ingredients, our model is fully deterministic, and displays quite different dynamics. Indeed, our model displays a rich array of different dynamic regimes, including the above development traps, but other regimes as well. The nature of the regimes, and the type of dynamic equilibrium depend on two sets of parameters, technology parameters and policy parameters. Let us examine these in turn.

There are two important technology parameters: the returns to scale for skilled workers and the degree of productive externalities between them, these two parameters being linked to the functional form of the production function (which will be developed in the section 3.1 below).

When positive externalities à la Romer (1990) dominate, we get the possibility of unstable equilibria, vicious circles, or virtuous circles, as we outlined above. If, however, the diminishing returns to scale dominate, the economy is much more stable and always converges to some sort of “Solowian” equilibrium.

So at this stage we would thus have, depending on the value of the above technology parameters, two types of economies: (a) Some “traditional” economies with a single long run equilibrium, towards which all dynamic trajectories converge. (b) Economies where this central equilibrium may be unstable, and development traps may occur.

However, the type of economies and dynamics are not only linked to technology parameters, but also to “policy parameters”. In our model, we define one such parameter, denoted $z$, which is meant to summarize all possible influences through which government can influence the growth of skilled workers.

A typical “academic” example of $z$ is the size of higher education in the country, and investment of government in education. An increase in that variable should normally increase the number of skilled workers. This paper shows that this policy variable has a substantial effect on the dynamics. The multiple equilibria with vicious and virtuous circles actually occur for

$^2$More precisely they obtain two different types of trajectories: “vicious circles” with high poverty and high brain drain, and “virtuous circles” with low poverty and low brain drain, between which the economy alternates stochastically.
median values of $z$. For low values of $z$, only the bad equilibrium, the “trap”, survives, whereas with a high value only the high equilibrium remains.

So our model displays a very rich array of dynamics, going from fully stable economy to multiple equilibria, development traps, vicious and virtuous cycles. We shall in the next sections go to a more formal presentation of the model and results.

The paper is divided into nine sections. We outline in section 2 some related literature. Section 3 describes the model. Section 4 studies the short run equilibrium. Section 5 describes the dynamics and long run equilibria. Section 6 begins, somewhat as a benchmark, with the traditional stable model. Section 7 studies the converse case, and shows when and how it can lead to development traps. Section 8 emphasizes the role of policy parameters. Section 9 concludes.

2 Related literature

The literature on “brain drain” has initially emphasized the negative effects of the flight of skilled workers. Early articles in this direction are notably Grubel and Scott (1966) and Bhagwati and Hamada (1974). Although we shall be ourselves also emphasizing the negative effects of brain drain, we must mention that lately a number of authors have shown that the possibility of migration could create some positive effects on the emigration country. This has been called a “brain gain” effect. This line of research has been studied by Mountford (1997), Stark, Helmenstein and Prskawetz (1997, 1998) and Stark (2004). Beine, Docquier and Rappoport (2001) and Easterly and Nyarko (2008) both derive the theoretical effects of migration on human capital creation, and test these effects empirically.

The literature on development traps is extremely vast, and builds on many different mechanisms. A wellknown contribution based on human capital accumulation is Azariadis and Drazen (1990).

There are very few papers linking the two issues of development traps and brain drain. We already described above De la Croix and Docquier (2010). Two other papers link migration and multiple equilibria, but in a different context than ours. Kwok and Leland (1982) have a model with multiple equilibria in migration, based on asymmetric information. Brézis and Krugman (1996) also present a multiple equilibria migration model, but where the focus is on the host country and not, as here, on the country of origin.
3 The model

As we emphasized above the two main building blocks of our model are the technology and population dynamics due to migration. We shall now describe both more formally.

3.1 Technology

Final output $Y_t$ consists of a homogeneous good, which is produced by competitive firms through the following Cobb-Douglas production function:

$$Y_t = AX_t^\gamma L_t^{1-\gamma}$$

where $A$ is an exogenous productivity parameter, $L_t$ unskilled labor and $X_t$ an aggregate intermediate good which, as we shall see, is produced with various types of skilled labor. The parameter $\gamma$ is an index of decreasing returns in $X_t$ and will play an important role below.

The variable $X_t$ is an index of the productivity of the skilled workers, which reflects some positive externalities between them in the tradition of Dixit and Stiglitz, 1977, Ethier, 1982, and Romer, 1990. More precisely assume that skilled workers work in intermediate industries indexed by $j \in [1, N_t]$, where $N_t$ is the number of intermediate goods. Then $X_t$ is given by the standard C.E.S. formula:

$$X_t = \left( \sum_{j=1}^{N_t} x_{jt}^\theta \right)^{1/\theta} \quad \theta < 1$$

The intermediate input $x_{jt}$ is produced by intermediate firm $j$ according to the production function:

$$h_{jt} = x_{jt} + f$$

where $h_{jt}$ is the amount of skilled labor employed in firm $j$ and $f$ is a fixed labor cost. These firms operate in a framework of monopolistic competition with free entry, through which the number $N_t$ will be determined endogenously, as we shall see below.

Finally we must express that the sum of skilled workers employed in all firms $j$ is equal to the aggregate amount of skilled labor $H_t$:

$$H_t = \sum_{j=1}^{N_t} h_{jt}$$
We should note at this stage that the production function (2) displays “returns to diversity”. This property appears clearly if we assume that all \( x_{jt} \) are equal to \( x_t \). Then (2) becomes:

\[
X_t = N_t^{1/\theta} x_t
\]  

(5)

Since \( \theta < 1 \), a higher number of intermediates \( N_t \) increases average productivity. The parameter \( \theta \) will be also important in the analysis below.

### 3.2 Population dynamics

We now turn to the dynamics of skilled and unskilled workers. Since the number of unskilled workers will play little role in what follows, we shall assume that it is constant in time:

\[
L_t = L \quad \forall t
\]  

(6)

It is assumed that the number of high skilled workers, \( H_t \), evolves according to:

\[
H_{t+1} = a (1 - e_t) H_t + z \quad a < 1 \quad z > 0
\]  

(7)

The first term in the right hand side represents the evolution of already existing skilled. First, there is a natural attrition rate \( a \) for skilled workers. Secondly, a fraction \( e_t \leq 1 \) of skilled workers emigrates between periods \( t \) and \( t+1 \). This is the “brain drain”, which we will study in detail in the next subsection.

The second term, \( z \), represents the influx of skilled workers not related to \( H_t \). This parameter should be thought of as influenced by government policy, in many possible different manners. For example immigration quotas for skilled labor, a tool used by many governments, will directly influence the influx. Also, a greater level of higher education in the country will typically increase the number of skilled workers. All these heterogeneous influences are subsumed in the simple “policy parameter” \( z \).

### 3.3 The migration function

For notational convenience, our working variable will not be the fraction of migrants \( e_t \), but rather \( s_t = 1 - e_t \), where \( s \) stands for “stayers”, i.e. the fraction of those skilled workers who stay in the country. We shall assume that \( s_t \) (and therefore \( e_t \)) is determined endogenously by:

\[
1 - e_t = s_t = S(\omega_t) \quad 0 \leq s_t \leq 1
\]  

(8)
where $\omega_t$ is the real wage of skilled workers at home. We assume:

$$S'(\omega_t) \geq 0$$

(9)

A typical function $S(\omega_t)$ is pictured in figure 1.

**Figure 1**

We will actually use in what follows migration functions of the form:

$$S(\omega_t) = \min (\lambda \omega_t^{\alpha}, 1) \ \ \ \alpha \leq 1$$

(10)

We shall now see that this possibility of a “brain drain” can modify dramatically the dynamics. We start by studying the short run equilibrium.

4 The short run equilibrium

4.1 The demand for intermediates

Let us denote as $P_t$ the price of output, $q_{jt}$ the price of intermediate good $j$ and $V_t$ the wage of unskilled labor. The objective of final output producing firms is to maximize profits:

$$P_t Y_t - \sum_j q_{jt} x_{jt} - V_t L_t$$

(11)

subject to the production functions (1) and (2). We can decompose the problem into two parts. First for any given value of $X_t$ firms will choose the amounts of intermediate goods $x_{jt}$ so as to minimize costs, i.e. so as to solve:

$$\text{Minimize } \sum_j q_{jt} x_{jt} \quad \text{s.t. } X_t = \left( \sum_{j=1}^{N_t} x_{jt}^\theta \right)^{1/\theta}$$

(12)

The first order conditions with respect to the $x_{jt}$'s are:

$$\frac{x_{jt}}{X_t} = \left( \frac{q_{jt}}{Q_t} \right)^{-1/(1-\theta)}$$

(13)

where $Q_t$ is the traditional C.E.S. aggregate index:

$$Q_t = \left( \sum_{j=1}^{N_t} q_{jt}^{\theta/(\theta-1)} \right)^{(\theta-1)/\theta}$$

(14)
The cost of $X_t$ is thus $Q_t X_t$. Now the level of $X_t$ will be chosen itself by maximizing profits:

$$P_t Y_t - Q_t X_t - V_t L_t$$

subject to production function (1), which yields the first order condition:

$$\frac{Q_t}{P_t} = \frac{\partial Y_t}{\partial X_t} = \gamma A X_t^{\gamma-1} L^{1-\gamma}$$

### 4.2 Monopolistic competition equilibrium

Let us now move to the intermediate firms $j$. We denote as $W_t$ the wage of the skilled workers. A monopolistically competitive firm producing intermediate good $j$ maximizes profits subject to the demand curve (13), i.e. it solves the program:

Maximize $q_{jt} x_{jt} - W_t h_{jt}$ s.t.

$$h_{jt} = x_{jt} + f$$

$$\frac{x_{jt}}{X_t} = \left(\frac{q_{jt}}{Q_t}\right)^{-1/(1-\theta)}$$

We find the following first order condition, traditional in monopolistic competition:

$$q_{jt} = \frac{W_t}{\theta} = q_t$$

The equilibrium is symmetrical, so that $x_{jt} = x_t$, $q_{jt} = q_t$, and:

$$X_t = N_t^{1/\theta} x_t \quad q_t = N_t^{(1-\theta)/\theta} Q_t$$

There is free entry into the intermediate firms’ industry, so the number of firms $N_t$ is determined by the zero profit condition for intermediate firms:

$$q_t x_t - W_t (x_t + f) = 0$$

Combining (3), (17) and (19) we obtain:

$$x_{jt} = x_t = \frac{\theta f}{1-\theta} \quad h_{jt} = h_t = \frac{f}{1-\theta}$$

and since $N_t (x_t + f) = N_t h_t = H_t$, we find the endogenous number of firms:
\[ N_t = \frac{(1 - \theta) H_t}{f} \]  

(21)

### 4.3 The real wage of skilled workers

Since this is a central element in the migration phenomenon, we now compute the real wage \( \omega_t \), i.e. the wage of skilled workers \( W_t \) deflated by the price of output \( P_t \):

\[ \omega_t = \frac{W_t}{P_t} \]  

(22)

We can decompose it as:

\[ \omega_t = \frac{W_t}{P_t} = \frac{W_t}{q_t} \frac{q_t}{P_t} \frac{Q_t}{Q_t} \]  

(23)

We know already from previous computations (formulas 16, 17 and 18):

\[ \frac{W_t}{q_t} = \theta \frac{q_t}{Q_t} = N_t^{(1-\theta)/\theta} \]  

(24)

\[ \frac{Q_t}{P_t} = \gamma AX_t^{\gamma-1} L^{1-\gamma} \]  

(25)

Combining (21), (23), (24) and (25), we find:

\[ \omega_t = \Lambda H_t^{(\gamma-\theta)/\theta} \]  

(26)

with:

\[ \Lambda = A \gamma \theta^{\gamma} L^{1-\gamma} h^{-\gamma(1-\theta)/\theta} \]  

(27)

We see that the real wage of skilled workers is increasing in \( H_t \) if \( \gamma > \theta \), i.e. if the effect of externalities dominates the diminishing returns in skilled labor. If in the contrary \( \gamma < \theta \), i.e. if diminishing returns dominate, then the real wage of skilled workers is decreasing in \( H_t \), the traditional result.

### 5 Dynamics and long run equilibria

Combining (7), (8), (10) and (26) we find that the dynamics of \( H_t \) is given by:

\[ H_{t+1} = a S (\omega_t) H_t + z = a \min (1, \xi H_t^\nu) H_t + z = F (H_t) \]  

(28)
with:
\[
\nu = \frac{\alpha (\gamma - \theta)}{\theta} \quad \xi = \lambda \Lambda^a
\] (29)

Long run equilibria will be solution of the equation:
\[
F (H_t) = a \min (1, \xi H_t^\nu) H_t + z = H_t
\] (30)

We see that the long run equilibria and the dynamics of equation (28) will be different depending on whether \( \nu \) is greater than or smaller than zero or, using equation (29), on whether \( \gamma \) is greater or smaller than \( \theta \). We start with the traditional case.

### 6 The traditional case

The “traditional” case is that where the diminishing returns to scale dominate the positive external effects, i.e. where:

\[
\gamma < \theta
\] (31)

We can first compute the derivative of the function \( F \):

\[
F' (H_t) = \frac{dH_{t+1}}{dH_t} = aS (\omega_t) \left[ 1 + \varepsilon (\omega_t) \frac{\partial \log \omega_t}{\partial \log H_t} \right]
\] (32)

where \( \varepsilon_t = \varepsilon (\omega_t) \) is the elasticity of \( S (\omega_t) \):

\[
\varepsilon_t = \varepsilon (\omega_t) = \frac{\partial \log S (\omega_t)}{\partial \log \omega_t} \geq 0
\] (33)

Further using formula (26), (32) becomes:

\[
F' (H_t) = aS (\omega_t) \left[ 1 + \varepsilon (\omega_t) \frac{\gamma - \theta}{\theta} \right]
\] (34)

Note that, from (10), \( \varepsilon (\omega_t) \leq 1 \). Since moreover \( \gamma < \theta \) and \( S (\omega_t) \leq 1 \), we have:

\[
0 \leq F' (H_t) \leq a < 1
\] (35)

The equation \( F (H_t) = H_t \) has a single long run equilibrium, which is dynamically stable. We denote it as \( H_S \) (\( S \) for stable). This case is represented in figure 2.

**Figure 2**
7 Development traps

Let us now assume that:

\[
\gamma > \theta
\]  

(36)
i.e. the positive externalities between skilled workers are dominant. In that case we shall now see that several patterns of development traps can occur.

It will be convenient for what follows to use the auxiliary function:

\[
G(H_t) = aH_t^{1+\nu} - H_t + z
\]  

(37)
The function \( G(H_t) \) starts at \( z \), and is convex since \( \nu > 0 \). One possibility is represented in figure 3.

Figure 3

The equilibria are the zero’s of the function:

\[
\min [G(H_t), z - (1 - a) H_t]
\]  

(38)

There are actually three distinct possibilities, which we now explore in turn.

7.1 Multiple equilibria (case A)

The first possibility, which corresponds to figure 3, is that where the initial “central” equilibrium, is unstable, so we denote it as \( H_U \). Two new stable equilibria, high and low, and denoted \( H_h \) and \( H_l \), now appear.

Figure 4

The dynamics of \( H_t \), which is represented in figure 4, displays multiple equilibria and strong history dependence. The actual path will fundamentally depend on whether the initial level of skilled workers \( H_0 \) is above or below \( H_U \).

1. If \( H_0 > H_U \), the high skilled workers mostly stay at home and \( H_t \) converges toward the high value \( H_h \).

2. If, however, \( H_0 < H_U \), the picture changes dramatically and we have a clear development trap, coming from the following vicious circle: a low initial \( H_t \) means a low \( \omega_t \). This triggers emigration of the skilled workers and therefore lowers \( H_t \), leading to lower \( \omega_t \) and so on. Finally the system will end up in the low value \( H_t \), the trap.
7.2 An unavoidable trap (case B)

But the matter can be even worse, as exemplified by the dynamics in figure 5. In such a case the high stable equilibrium $H_h$ and the unstable intermediate equilibrium $H_U$ have disappeared, and all that remains is the low equilibrium $H_l$. Whatever the starting point $H_0$, $H_t$ will converge toward this low trap value.

Figure 5

7.3 Escaping the trap (case C)

The last possibility is actually the most favorable of the three. Although the combination of parameters $\gamma$ and $\theta$ is consistent with development traps, we see that this time it is the low stable and middle unstable equilibria, $H_l$ and $H_U$, that have disappeared, so that the economy will always converge toward a “high” equilibrium $H_h$. This is represented in figure 6.

Figure 6

8 Policy and traps

We shall investigate here a little further the role of the “policy parameter” $z$ in producing or not development trap dynamics.

We saw in section 7 that for $\gamma > \theta$ the economy becomes prone to traps. We could distinguish, however, in such a case three different dynamic regimes (cases $A$, $B$ and $C$ above). We shall now show that the value of the “policy parameter” $z$ will be instrumental in determining which of the three regimes the economy is in.

8.1 Parameters and regimes

Consider the equation $G(H_t) = z - (1 - a)H_t$ (cf figure 3), and define as $H^*$ the (nonzero) root of that equation. Using figure 3, associated to case $A$, and the corresponding figures for cases $B$ and $C$, we see that:

(a) Case $A$ will occur if $G(H^*) > 0$ and the equation $G(H_t) = 0$ has two real roots.

(b) Case $B$ will occur if $G(H^*) < 0$ and the equation $G(H_t) = 0$ has two real roots.

(c) Case $C$ will occur if the equation $G(H_t) = 0$ has two complex roots.
Using the above expression of the function $G$, (equation 37), we see that the above conditions (a), (b) and (c) can be expressed in terms of the original parameters of the model. For that we use the following two threshold values for $z$:

$$z_1 = (1 - a) \left( \frac{1}{\xi} \right)^{1/\nu}$$  \hspace{1cm} (39)

$$z_2 = \nu \left( \frac{1}{a \xi} \right)^{1/\nu} \left( \frac{1}{1 + \nu} \right)^{(1+\nu)/\nu}$$  \hspace{1cm} (40)

One can compute that for all values of the parameters:

$$z_1 \leq z_2$$  \hspace{1cm} (41)

Accordingly the separation conditions (a), (b) and (c) just above translate into:

(a) Case $A$ occurs if $z_1 < z < z_2$.
(b) Case $B$ occurs if $z < z_1$.
(c) Case $C$ occurs if $z > z_2$.

So we see that, when $\gamma > \theta$, the policy parameter $z$ is fundamental in determining, among the three cases $A$, $B$ or $C$, the exact nature of “trap dynamics”.

9 Conclusion

This paper has developed a simple model where brain drain can lead to very serious development traps, even though the same economy without workers’ mobility would be stable.

Inspite of its simplicity this model displays an extremely rich array of dynamic regimes, including a “Solowian” one, vicious and virtuous circles, multiple equilibria and also cases where the economy can be trapped in a single low or high state.

The factors leading to one regime or the other are, as one would expect, some technological factors (productive returns and externalities). But it also turns out that, in the regimes prone to traps, a government policy variable can be powerful in orienting the economy towards “good” or “bad” outcomes.
References


Figure 1

$S(\omega_t)$

$\omega_t$

1
Figure 2
$G(H_t)$

Figure 3
Figure 4
Figure 5

\[ H_{t+1} = F(H_t) \]

\[ H_{t+1} = H_t \]
Figure 6

\[ H_{t+1} = H_t \]

\[ H_{t+1} = F(H_t) \]