Dynamic brand-image-based production location decisions

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Abstract

In this paper, we study the dynamic production location decisions of a manufacturer of a certain branded product. Considering brand-image as a form of goodwill, we extend the well-known Nerlove–Arrow dynamic model by adding both country-image and price. Formulating an optimal control problem for a group of countries in which the cost of production is convexly increasing with country-image, we are able to develop optimal decision rules for a manufacturer regarding the location of production and pricing over time. The resulted optimal policy has a very interesting pattern. Assuming that the demand rises by more than the value of the new brand-image in percentage terms, then, if brand-image is increasing toward a stationary value level, the optimal policy should be to initially locate production in countries with high image and set a high price that signals high quality. Later, the production should gradually shift to countries with lower production costs and lower image and the price lowered until the stationary value level is reached. For brand-images beyond the stationary value level, the location of production should start in a country with low costs and country-image while setting prices that signal relatively low quality. Over time, production should be shifted to countries with gradually higher costs and images while setting higher prices until the brand-image approaches the level of stationary value.

Keywords: Country-of-origin; Manufacturing country; Production sourcing; Brand-image; Price-quality; Country-image; Goodwill; Optimal control; Decision-rules

1. Introduction

Brand-image is defined as perceptions about a brand as reflected by the brand associations (attributes, benefits and overall brand attitudes) held in consumer memory (Keller, 1993). Country-image is the total of all descriptive, inferential and informational beliefs that a consumer has about a particular country (Martin & Eroglu, 1993). Both current and past images of sourcing countries play a role in determining brand and product perceptions. For example, the perceived image of a product made in a country having a strong image (USA) and brand-image (GE) may deteriorate by sourcing production in countries with weak images such as the emerging economies in Eastern Europe (cf. Brodowsky, Tan, & Meilich, 2004; Nebenzahl & Jaffe, 1996). Selecting a country of manufacture has become a critical decision variable for managers of global companies. They may decide to design a product in one country and manufacture it in another. Their decision may be based on cost considerations or proximity to end user markets, but country-image has also now become a major managerial decision variable (Brodowsky et al., 2004). The influence of current and future country-images on production sourcing decisions is the focus of this work. The sourcing country may improve or erode brand-image and consequently sales. Reducing costs of production by means of sourcing may improve profits. If the low cost is associated with a weak-image of the sourcing country, the erosion of brand-image may have a greater effect and thus reduce profits. However this is not always the case. For example,
strong Japanese brands such as Sony, which have developed a substantial brand reputation, have been able to overcome such stereotyping, at least sufficiently enough to compete with Korean or Taiwanese firms, despite having a “made in” label of a Southeast Asian country such as Malaysia (Choi, 1992).

Previously, the location of production has been generally viewed as a function of transaction costs, country-specific attributes such as cost of raw materials and wage rates, negotiating and monitoring costs, environmental considerations such as political risk and competition, and strategic variables such as first mover advantages. Thus, in most, if not all production location studies, the focus have been exclusively on the supply side, ignoring the effect location has on the demand side, that is, on brand-image and its implications (Li, Murray, & Scott, 2000). In this paper, we examine location of production as a function of costs, of country and brand-images, as well as the short and long-run effects of price. We consider a monopolist manufacturer that needs to decide where to locate the production of a certain branded product. We posit that the perceived quality of a given product is based on its brand-image, country-image and price, all of which determine the demand for the particular product (cf. Darling & Arnold, 1988; Hastak & Hong, 1991; Teas & Agarwal, 2000; Thorelli, Lim, & Ye, 1989). Considering brand-image as a form of goodwill, we extend the well-known Nerlove–Arrow dynamic model by adding both country-image and price effects. Formulating an optimal control problem for a group of countries in which the cost of production is convexly increasing with country-image, we are able to develop optimal decision rules for a manufacturer regarding the location of production and pricing over time. The resulted optimal policy has a very interesting pattern. Assuming that the demand rises by more than the value of the new brand-image in percentage terms, then, if brand-image is increasing toward a stationary value level, the optimal policy should be to initially locate production in countries with high image and set a high price that signals high quality. Later, the production should gradually shift to countries with lower production costs and lower image and the price lowered until the stationary value level is reached. For brand-images beyond the stationary value level, the location of production should start in a country with low costs and country-image while setting prices that signal relatively low quality. Over time, production should be shifted to countries with gradually higher costs and images while setting higher prices until the brand-image approaches the level of stationary value.

The rest of this paper is organized as follows. In Section 2, we present our model. In Section 3, an optimal production policy is determined based on the current brand-image and on model’s parameters. In Section 4, conclusions and managerial implications of the model are discussed. In Section 5, we present our conclusions and future research directions.

2. Model formulation and notation

We consider a monopolist manufacturer that needs to decide in which country to produce a certain branded product. Each potential country has a certain image and cost of production. The image of the country can improve or impair the manufacturer’s brand-image; the better the country-image, the higher the brand-image and vice versa (Papadopoulos, Heslop, & Avlontis, 1987). Since consumers tend to associate high prices with high quality, the price assigned to the product has similar long-run effects on brand-image, where higher prices improve the long-run brand-image. The brand-image resulting from both effects directly impacts the sales of the particular product. In addition, following the classical demand function, price is negatively related to sales in the short run. Thus, price has both long-run and short-run effects on sales. On the other hand, the cost of production negatively affects the resulted profits from the sales of the branded product. We summarize these relationships in a block diagram described in Fig. 1. To better understand the problem, and eventually select a target country in which to locate production, we present a formal decision-making model below.

Let \( x = x(t) \) be the image of the brand at time \( t \). This image summarizes the previously perceived quality of similar products sold under the same brand name, as well as all past effects of countries of production where similar branded products were produced and of past prices of such products. This can be thought as accumulated goodwill of the brand at time \( t \).

Let \( p = p(t) \) be the price of the brand at time \( t \). There is ample evidence in the literature that consumers perceive high prices as indicators of high quality, and thereby, brand reputation (Feichtinger, Luhmer, & Sorger, 1988; Kotowitz & Mathewson, 1979a, 1979b; Spremann, 1985). For good reviews of this topic see Sethi (1977) and Feichtinger, Hartl, and Sethi (1994). Following this approach and noting that brand-image may be considered as a measure of reputation, we relate price with brand-image. Based on past experience, consumers form a price expectation for every brand within a given product line. We argue that there is a certain range of prices in which a decrease in price increases sales according to the classical demand function. However, a decrease under the expected price \( \bar{p} \) signals low quality and, thereby, low brand-image. It should be noted that \( \bar{p} \) is a brand attribute. Consumers expect different
prices for different brands of products within the same product line.

Finally, let \( u = u(t) \) be the country-image level of a country as a source for a considered line of products at time \( t \). As mentioned above, brand-image improves or worsens with the effect of country-image. To incorporate the price and country-image effects on brand-image we extend the well-known Nerlove and Arrow (1962) model to the following dynamic equation:

\[
\dot{x}(t) = k_1(p(t) - \bar{p}) + k_2u(t) - \delta x(t), \quad x(0) = x_0. \tag{2.1}
\]

The left-hand side of Eq. (2.1) represents the change in brand-image. The first term on the right-hand side represents the impact of price levels that are above or under a certain expected price for the brand, \( \bar{p} \), on the brand-image. A price above \( \bar{p} \) has a positive impact on the brand-image, imparting the perception of improved quality, and below it has a negative impact, implying deterioration in quality. The second term in (2.1) represents the impact of the country-image level on the brand-image level. The third term reflects the rate at which the brand-image deteriorates. This term may be taken as capturing the effect of all competitors’ actions on the image of the brand, making the assumption of a monopolist producer more reasonable.\(^1\) We take \( k_1 \geq 0 \) as representing the price effect on the brand-image, \( k_2 > 0 \) as representing the country-image effect on brand-image and \( \delta > 0 \) as representing the brand-image deterioration rate. The constants \( k_1, k_2, \delta \) and \( \bar{p} \) can be empirically determined. Note that the case \( k_1 = 0 \) and \( k_2 = 1 \) mathematically corresponds to the Nerlove–Arrow (1962) model. For given \( k_1, k_2, \delta \) and \( \bar{p} \), (2.1) states that a change in brand-image will be positive (negative) if the combined impact on the brand-image of current price and country-image, \( k_1(p(t) - \bar{p}) + k_2u(t) \), will be higher (lower) than the brand-image depreciation, \( \delta x(t) \).

To formulate the problem for the monopolistic firm, assume that the rate of sales \( S = S(t) \) is governed by a long-run component (brand-image), \( x \), and a short-run component (price), \( p \), as

\[
S = S(p, x). \tag{2.2}
\]

We assume that the rate of sales decreases with the price but increases with the brand-image. This leads to the following conditions on the first partial derivatives of \( S \):

\[
\partial S/\partial p < 0 \quad \text{and} \quad \partial S/\partial x > 0. \tag{2.2a}
\]

Let \( c = c(t) \) be the production cost per unit of sales in a certain country. For most manufactured consumer products, the more developed a country, the higher its image as a sourcing location. Moreover, the more developed is a country, the higher its standard of living and the higher the costs of factors of production, and in particular, the cost of labor. Thus, one can expect a high correlation between country-image and production costs.

\[\text{Table 1}\]

<table>
<thead>
<tr>
<th>Country</th>
<th>CPI</th>
<th>Wage rates$^\text{a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S. Korea</td>
<td>4.05</td>
<td>43</td>
</tr>
<tr>
<td>Israel</td>
<td>4.59</td>
<td>66</td>
</tr>
<tr>
<td>France</td>
<td>4.77</td>
<td>94</td>
</tr>
<tr>
<td>Canada</td>
<td>4.92</td>
<td>90</td>
</tr>
<tr>
<td>Japan</td>
<td>5.13</td>
<td>106</td>
</tr>
<tr>
<td>USA</td>
<td>5.13</td>
<td>100</td>
</tr>
<tr>
<td>Germany</td>
<td>5.15</td>
<td>144</td>
</tr>
</tbody>
</table>

\(^1\) Source: Derived from data collected in 1997 for a multinational study (Nebenzahl et al., 2003).

\(^2\) Source: Derived from 1997 data (United States Department of Labor, 2004).

Indeed, there is a relationship between country-image perceptions, as measured by a country-image scale (Nebenzahl, Jaffe, & Usunier, 2003), and cost of production (using wage rates as a proxy) as shown in Table 1 for 1997. For example, in high-country-image countries, e.g. the United States, Japan and Germany, wage rates are high. In lower image countries, e.g. South Korea and Israel, wage rates are lower.

Accordingly, we assume that \( c \) depends on the country-image \( u(t) \), thus \( c(u) \). We assume that this relation is valid in a group of countries with country-image values in the interval \( U = [u_L, u_H] \), where \( U \) is the scale on which country-image is measured,\(^2\) and \( u_L \) and \( u_H \) represent the lowest and highest country-images in this scale. This observation allows us to merge the revenue with the cost effects of the sourcing decision into a net profit equation, \( R \), (total revenue net of production costs),

\[
R(p, u, x) = [p - c(u)]S(p, x). \tag{2.3}
\]

Given the positive correlation between country-image and cost of production, we expect \( c \) to be an increasing function of \( u \), thus,

\[
c'(u) > 0. \tag{2.4}
\]

Assuming that the manufacturer maximizes the present value of net profit streams discounted at a fixed discount rate \( r \), with respect to price and country-image, over an infinite horizon, the manufacturer’s problem becomes

\[
\begin{align*}
\max_{p, u} & \quad \left[ J = \int_0^\infty R(p, u, x)e^{-rt} \, dt \right] \\
\text{s.t.} & \quad \dot{x} = k_1(p - \bar{p}) + k_2u - \delta x, \quad x(0) = x_0.
\end{align*} \tag{2.5}
\]

where \( u \in U \), and \( R \) is as in (2.3). In the control theory framework, \( u \) and \( p \) are control variables and \( x \) is a state variable.

\[\text{Note that the case was utilized in Table 1.}\]

\[\text{For a recently developed scale, see Nebenzahl et al. (2003). This scale was utilized in Table 1.}\]
3. Optimal policy

We use Pontryagin’s maximum principle (Pontryagin, Boltyanskii, Gamkrelidze, & Mishchenko, 1962) to find the optimal solution. We form the current-value Hamiltonian (Arrow & Kurtz, 1971)

\[ H = (p - c(u))S(p, x) + \lambda [k(p - \bar{p}) + k_2u - \delta x]. \]  

(3.1)

The current-value adjoint variable \( \lambda \) satisfying the differential equation

\[ \dot{\lambda} = r\lambda - \frac{\partial H}{\partial x} = (r + \delta)\lambda - [p - c(u)] \frac{\partial S}{\partial x} \]  

(3.2)

and the condition that

\[ \lim_{t \to \infty} e^{-rt}\lambda(t) = 0. \]  

(3.3)

The adjoint variable \( \lambda(t) \) is the shadow price associated with the brand-image at time \( t \). The differential equation (3.2) tells us what is the value of increasing the brand-image at each point in time. Thus, the Hamiltonian in (3.1) can be interpreted as the instantaneous profit rate which includes the value \( \lambda \dot{x} \) of the new brand-image \( x \) created by the effect of price relative to the expected price, \( (p - \bar{p}) \), and the country-image \( u \).

Eq. (3.2) corresponds to the equilibrium relation for investment in brand-image (compare with capital goods in Arrow & Kurtz, 1971). In our context, the equilibrium is in relation to the image implied by the brand’s price and the country selected for production. It states that the marginal opportunity cost \((r + \delta)\lambda \) of investment in brand-image should equal the marginal profit \( (p - c(u)) \frac{\partial S}{\partial x} \) from increased brand-image and the image gain \( \lambda \).

The necessary first-order optimality conditions are \( \partial H/\partial p = 0 \) and \( \partial H/\partial u = 0 \). Thus

\[ S + (p - c(u)) \frac{\partial S}{\partial p} \dot{\lambda} + k_1 = 0 \]  

(3.4)

and

\[ -c'(u)S + \dot{k}_2 = 0. \]  

(3.5)

Considering (2.4) and the fact that \( S > 0 \) and \( k_2 > 0 \), we obtain that

\[ \dot{\lambda} > 0. \]  

(3.5a)

Condition (3.5a) is consistent with the intuition that increasing the brand-image by one unit has a positive value.

Note that \( \eta = -(p/S) \frac{\partial S}{\partial p} \) is the elasticity of demand with respect to price\(^3\) so (3.4) can be written as

\[ p^* = \frac{\eta}{\eta - 1} \left( c(u) - \frac{\dot{k}_1}{\frac{\partial S}{\partial p}} \right), \]  

(3.6)

where \( p^* \) denotes the optimal pricing policy. If the firms ignore the long-run effects of price and country-image on brand-image (thus \( \lambda = 0 \)) then (3.6) is the usual price formula for the monopolist. We call this phenomenon a myopic price. Thus, the myopic price is the usual monopolist pricing decision rule.

Considering (3.5a) and (3.6) we conclude with the following result.

**Proposition 1.** If the production cost increases with the country-image then the optimal brand’s price is always above the myopic price. Formally,

\[ p^* > \frac{\eta}{\eta - 1} c(u). \]  

(3.7)

**Proof.** See Appendix.

Proposition 1 states that a brand’s price taking into account the price and country-image effects on brand-image will be higher than the myopic price. This result is consistent with a brand differentiation policy that facilitates higher prices.

**Optimal brand-image:** Defining \( \beta = (x/S) \frac{\partial S}{\partial x} \) as elasticity of demand with respect to brand-image and using (3.2), we can derive

\[ x^* = \frac{\beta(p^* - c(u))S}{(r + \delta)\lambda - \lambda}. \]  

(3.8)

The interpretation of (3.8) is that at optimum, the ratio of brand-image to profit, is directly proportional to the brand-image elasticity and inversely proportional to the sum of the marginal opportunity cost \((r + \delta)\lambda \) of investment and the rate at which the potential contribution of a unit of brand-image to profits becomes its past contribution \((-\dot{\lambda})\).

**Sufficient conditions:** As mentioned, Eqs. (3.4) and (3.5) are the necessary first-order optimality conditions for \( p \) and \( u \), respectively. In addition to these conditions, we must verify second-order conditions to ensure that we are indeed maximizing profits. Such a sufficient condition for local optimum (see Bryson & Ho, 1975) is that the Hessian matrix of \( H \), denoted by \( \hat{H} \), is negative definite,\(^4\)

\[ \hat{H} = \begin{bmatrix} 2S_p + |p - c(u)|S_{pp} & -c'(u)S_p \\ -c'(u)S_p & -c''(u)S \end{bmatrix} < 0, \]  

(3.9)

for all \((p, u)\) in the neighborhood of the optimal policy, \((p^*, u^*)\). In (3.9) \( S_p \) and \( S_{pp} \) denote the first- and second-order partial derivatives of \( S \) with respect to \( p \).

Considering (2.4) and (3.9) we conclude that the function \( c(u) \) should increase convexly (see footnote 4). Note that the countries included in Table 1 satisfy this condition.

\(^3\) As common in the pricing literature (e.g. Kalish, 1983), it is assumed that \( \eta > 1 \).

\(^4\) This requires that the principal minors of matrix \( \hat{H} \) alternate in sign, beginning with negative. This implies: \( 2S_p + (p - c(u))S_{pp} < 0 \) and \( c''(u) > 0 \).
3.1. The feedback solution

A solution that is specified as depending on the state variable \( x \), the brand-image, rather than directly on time is referred to as a feedback solution. This is the type of solution we seek. Thus, we solve for \( p^* \) and \( u^* \) as functions of the state variable \( x \). Note that because feedback solutions depend on the state variable, they are more realistic than solutions that are only functions of time and that do not get revised if during some time interval the brand-image value increases more or less than the expected amount. Feedback solutions are therefore more attractive and useful to managers. However, feedback solutions are generally mathematically more difficult to obtain. Fortunately, we have been able to find the feedback solution for this problem, and we will use it to compute the optimal policy. The following theorem helps us to compute the feedback optimal policy.

**Theorem 1.** Consider the manufacturer’s optimization problem (2.5). If condition (3.9) holds in some neighborhood of \((p^*, u^*)\), then (3.4) and (3.5) have a unique local optimal time-invariant feedback solution for the pair \((p^*, u^*)\), of the following form:

\[
p^* = p^*(x, \Phi(x)) \quad \text{and} \quad u^* = u^*(x, \Phi(x)).
\]

(3.10)

The function \( \Phi(x) \) is the unique solution of the following backward differential equation:

\[
\Phi'(x)[k_1(p^* - \bar{p}) + k_2 u^* - \delta x]
= (r + \delta)\Phi(x) - (p^* - c(u^*)) \frac{\partial S(p^*, u^*)}{\partial x},
\]

\[
\Phi(\bar{x}) = \bar{\lambda}.
\]

(3.11)

in the neighborhood of \( \bar{x} \), where \( \bar{x} \) and \( \bar{\lambda} \) are the long-run stationary equilibrium values of the state and co-state.\(^5\)

**Proof.** See Appendix.

The solution based on Theorem 1 is made clear in the next sub-section and in the Illustrative Examples sub-section. Note that Theorem 1 guarantees the existence of the feedback solution only locally around the steady state. However, this feedback solution exists globally by the following argument. Given the solution of the optimal control problem is unique the state has to evolve monotonically (Hartl, 1987), there exists a one-to-one relationship between the time and state. Therefore, for each state on the solution path there is a unique time point and one can, therefore, write the co-state as function of the state rather than of time.

The marketing decision model that uses the rules in (3.10) acts as a closed-loop system, see Fig. 2. The firm’s marketing strategies on choosing the sourcing country-image and brand’s price drive the dynamic marketing response model leading to the new brand-image output. The new output is considered in the evaluations of the optimal decisions to maximize profits (the decision rules (3.10)) that are the controllable inputs of this system or the brand-image-based inputs.

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\(^5\) \( \bar{x} \) and \( \bar{\lambda} \) are solutions of \( \dot{x} = 0 \) and \( \dot{\lambda} = 0 \).
is the following. If demand increases by more than the value of the new brand-image \( \dot{x} \) (created by the effect of price relative to the expected price and the country-image), then the firm should make profits through cost efficiencies and locate production based on cost considerations only. An example of this strategy is Sony producing TV sets in Malaysia rather than in Japan. If, on the other hand, the value of the new brand-image increases by more than the demand then the firm should make profits from the brand-image improvements by producing in countries with a high image. For example, the Chinese appliance manufacturer Haier located refrigerator production in the United States in order to improve its brand-image. Of course, this strategy also reduced transaction costs of shipping the finished product from China.

### 3.2. A multiplicative price and brand-image response function

Our results so far are based on general functional forms. We can get a better insight into the optimal policy by examining specific functional forms that satisfy the assumptions of our model. Our specification for the sales response \( S \) is the multiplicative price and brand-image function, thus

\[
S = S(x, p) = ap^{-\eta}x^\beta.
\]  
(3.14)

Considering (3.14) we conclude with the following result.

**Proposition 3.** For a multiplicative price and brand-image response function, and cost of production increasing convexly with the country-image, the brand’s optimal price increases with the country-image.

**Proof.** See Appendix.

This proposition is intuitive, namely that higher brand-images resulting from higher country-images allows the firm to charge premium prices.

Considering Propositions 2 and 3 we conclude with the following result:

**Proposition 4.** For a multiplicative price and brand-image response function, and cost of production increasing convexly with the country-image, the brand’s optimal price decreases with an increase of the brand-image if and only if condition (3.13) holds.

**Proof.** See Appendix.

According to Proposition 4, if (3.13) holds, the brand’s price should decrease with the increase in brand-image. If, on the other hand, (3.13) does not hold, the brand’s price should increase with the increase in brand-image. The implication of Proposition 4 is the following. If the demand rises by more than the value of the new brand-image \( \dot{x} \) then the firm should make a profit through the short-run effect of an increase in sales by reducing prices. If the value of the new brand-image rises by more than demand, then the firm should make a profit through the long-run effect of an increase in price as the brand-image increases.

Combining Propositions 2 and 4, when demand increases by more than the value of the brand-image, the normative rule of a profit-maximizing manufacturer when the brand-image improves is to reduce the brand’s price and select a country with lower image. Thus, the normative rule suggests that, in this case, the profit-maximizer should take advantage of short-run rather than long-run effects and increase profits through reducing prices and costs (by choosing the country with lower production costs and lower country-image). However, if the value of the brand-image created by long-run effects increases by more than the demand, then the normative rule suggests that when the brand-image improves the profit-maximizer should take advantage of long-run rather than the short-run effects, and increase profits by increasing prices as the brand-image increases (by choosing a country with a higher, positive country-image).

**The feedback solutions of Theorem 1:** Considering Theorem 1, (3.14) and (3.4) we obtain (for details, see the proof of Proposition 3, in Appendix),

\[
p = \frac{\eta}{(\eta - 1)} c(u) \frac{k_2(\eta - 1)}{k_2(\eta - 1) - k_1 c(u)}.
\]  
(3.15)

Substituting (3.15) into (3.5), gives us an equation in \( u \), \( \Phi(x) \) and \( x \).

\[
-c'(u) a \left[ \frac{k_2(\eta - 1) - k_1 c'(u)}{k_2 c(u)} \right]^{\eta} + k_2 \Phi(x) x^{-\beta} = 0.
\]  
(3.16)

For a specification of \( c(u) \) and given \( \eta \) and \( \beta \) one can solve Eq. (3.16) for a feedback solution \( u^* = u^*(x, \Phi(x)) \). Substituting in (3.15) we have \( p^* = p^*(x, \Phi(x)) \). Substituting these solutions in (3.11), we solve for \( \Phi(x) \) and get our solutions as a function of the brand-image \( x \).

Considering (3.14) and (3.16) we conclude with the following result.

**Proposition 5.** For a multiplicative price and brand-image response function, and cost of production increasing convexly with the country-image, a necessary condition for the brand’s optimal price to increase,

(i) with the effect of the price on brand-image, \( k_1 \), and,  
(ii) with the effect of country-image on the brand-image, \( k_2 \), is that

\[
c'(u) < \frac{(\eta - 1) k_2}{(\eta + 1) k_1}.
\]  
(3.17)

**Proof.** See Appendix.

Note that condition (3.17) is necessary but not sufficient (see the proof).

To conclude, price has both long-run image and short-run demand effects, while country-image has only long-run effects. \( k_1 \) determines the magnitude of the long-run effect of the brand’s price relative to its expected price while \( k_2 \) is the long-run effect...
of country-image. According to Proposition 5, for a stronger effect of long-run price and country-image to allow a higher price it is necessary that condition (3.17) will be satisfied.

To illustrate our solutions and get more insights we consider next some illustrative examples.

3.3. Illustrative examples

We consider the following specification for the cost of production,

\[ c(u) = b + du^2. \]  

(3.18)

For the purpose of illustration, we chose for the parameters to be the following:

\[ a = 1, \quad \eta = 2, \quad \beta = 1 \]

(parameters for the price and brand-image response function in (3.14)),

\[ \hat{p} = 3, \quad k_1 = .01, \quad k_2 = 1, \quad \delta = .01 \]

(cost parameters and discount rate).

We solved for the optimal feedback policy for this problem, as displayed in Fig. 3.

Characterization of the optimal policy: As can be seen in Fig. 3, as long as \( p^*(x) > p_{ss} \), where \( p_{ss} \) represents the stationary value of the price (steady state), \( p^* \) decreases as \( x \) increases (the arrows in Fig. 3 indicate the direction of the time). On the other hand, if \( p^*(x) < p_{ss} \), \( p^* \) increases as \( x \) decreases. Furthermore, if \( p^*(x) > p_{ss} \), \( p^* \) decreases much faster to the value \( p_{ss} \) than it increases to \( p_{ss} \) when \( p^*(x) < p_{ss} \). Thus, the optimal policy for lower prices than \( p_{ss} \) is to increase slowly to \( p_{ss} \) as the brand-image depreciates to the stationary value \( \bar{x} \). The optimal policy for higher prices than \( p_{ss} \) is to decrease quickly to \( p_{ss} \). A similar feature occurs for the country-image. When \( x > \bar{x} \) the country-image \( u^* \) takes low values and increases slowly until the brand-image depreciates to the level \( \bar{x} \) (the stationary value) at which \( u^* = u_{ss} \) and stays at this level to sustain the brand-image at \( \bar{x} \). When \( x < \bar{x} \) the country-image \( u^* \) starts at higher values and decreases quickly as \( x \) increases to \( \bar{x} \) at which \( u^* \) takes the value \( u_{ss} \). This type of behavior is akin to goodwill according to the Nerlove–Arrow (1962) model. Note that the case solved here satisfies condition (3.13), and the solutions follow the pricing and country-image decision rules that result from Propositions 2 and 4.

4. Conclusions and managerial implications

Consumer perception of a country’s image and product price are among the key variables that determine brand-image. A more positive country of manufacture image generally results in a more positive brand-image. This leads to more sales and thus more revenues. However, more positive image countries generally have higher production costs, thus profits may be lower, unless the higher country-image can allow for a higher finished product price and brand-image. Thus, the dilemma for a sourcing company is how to optimize the tradeoff between country-image and cost. In this paper, we present an analytic model to deal with this dilemma. We find decision rules for production location based on current brand-image of a product.

The optimal decision rules have a very interesting pattern. Assume that the demand rises by more than the value of the new brand-image in percentage terms. If brand-image is increasing toward the stationary value, the optimal policy should be to initially locate production in countries with high image and set a price that signals high quality. Then, over time, gradually shift sourcing to lower production costs and lower image countries, while lowering the product’s price. The intuition behind this is that a high-origin country-image will allow building a strong brand-image, and this may offset the host country’s lower image when production is shifted to a lower image lower cost country. Thus, the high-origin country-image will enable the firm to maintain its high-quality brand-image until the brand reaches the stationary value point. If, for example,
Appendix

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Appendix

Proof of Proposition 1. The proposition follows by considering (3.6) and (3.5a) and the fact that \( \partial S/\partial p < 0 \). \( \square \)

Proof of Theorem 1. Under the assumption of the theorem we may apply the implicit function theorem, see Apostol (1979, p. 374), to Eqs. (3.4) and (3.5) in the unknowns \( p \) and \( u \). Thus, we conclude with the solutions

\[
p^* = p^*(x, \lambda) \quad \text{and} \quad u^* = u^*(x, \lambda).
\]  

(A.1)

Let

\[
\lambda = \Phi(x).
\]  

(A.2)

Now, the theorem follows immediately by substituting (A.2) and its derivative with respect to \( t \) into (A.1) and (3.2), and considering the relation \( \lambda = \Phi(x) \hat{\lambda} \) and the existence and uniqueness theorem of differential equations (see Boyce & Diprima, 1986). \( \square \)

Proof of Proposition 2. Taking the derivative of (3.5) with respect to \( x \), we obtain

\[
\partial u/\partial x = [-c'(u) \partial S/\partial x + k_2 \Phi'(x)]/(S \partial c''(u)). \]  

(A.3)

Since \( c''(u) > 0 \), the sign of \( u_x \) is as the sign of the numerator of (A.3). Thus, \( u_x < 0 \) if and only if

\[
c'(u) > k_2 \Phi'(x)/S_x.
\]  

(A.3a)

Considering again (3.5), \( c'(u) = \Phi(x)k_2/S \), thus (A.3a) is equivalent with

\[
\partial S/\partial x > \Phi'(x)/\Phi(x).
\]  

\( \square \)

Proof of Proposition 3. Considering (3.5) and Theorem 1 we have

\[
\Phi(x) = c'(u)S/k_2.
\]  

(A.4)

Substituting (A.4) into (3.4) and considering (3.14) we obtain

\[
p = \frac{\eta}{(\eta - 1)} \frac{c(u)}{k_2(\eta - 1) - k_1 c'(u)}.
\]  

(A.5)

Taking the derivative (A.5) with respect to \( u \) we obtain

\[
p'(u) = k_2 \eta^2 \frac{c'(u)[k_2(\eta - 1) - k_1 c'(u)] + k_1 c''(u)c(u)}{[k_2(\eta - 1) - k_1 c'(u)]^2}.
\]  

(A.6)

Comparing (3.7) and (A.5), we obtain

\[
k_2(\eta - 1) - k_1 c'(u) > 0.
\]  

(A.7)

Now, the proposition follows from (A.6) and (A.7) and the assumptions that \( c'(u) \) and \( c''(u) \) are positive. \( \square \)

Proof of Proposition 4. Considering (A.5) we obtain

\[
\partial p/\partial x = p'(u) \partial u/\partial x,
\]  

(A.8)

where \( p'(u) \) is as in (A.6). Now, the proposition follows from Propositions 2 and 3. \( \square \)

Proof of Proposition 5. Part (i): Considering (3.15) and taking the derivative (total) of \( p \) with respect to \( k_1 \) we obtain

\[
\frac{dp}{dk_1} = p'(u) \frac{\partial u}{\partial k_1} + \frac{\partial p}{\partial k_1},
\]  

(A.9)
Thus, considering (A.7), a necessary condition that (3.16) and denote
\[
g(u, k_1, k_2, x) = -c'(u)\left(\frac{k_2(\eta - 1) - k_1c'(u)}{k_2\eta c(u)}\right)\eta
+ k_2\phi(x)x^{-\beta} = 0. \tag{A.10}
\]
To find \(\partial u / \partial k_1\) we take the derivative of the implicit function \(g(u, k_1, k_2, x)\) with respect to \(k_1\), we obtain
\[
\frac{\partial g}{\partial u} \frac{\partial u}{\partial k_1} + \frac{\partial g}{\partial k_1} = 0.
\]
Thus,
\[
\frac{\partial u}{\partial k_1} = -\frac{\partial g}{\partial u} \frac{\partial g}{\partial k_1}. \tag{A.11}
\]
Considering (A.10) we obtain
\[
\frac{\partial g}{\partial k_1} = \frac{ac'(u)^2\left(\eta k_2 c(u)\right)^{1-\eta}}{k_2 c(u)} \tag{A.11a}
\]
and
\[
\frac{\partial g}{\partial u} = \frac{a}{k_2(\eta - 1) - k_1c'(u)} \left(\eta k_2 c(u)^{1-\eta} - [\eta c'(u)^2 - c(u)c''(u)][k_2(\eta - 1) - k_1c'(u)] + \eta k_1 c(u)c'(u)c''(u))\right) / k_2\eta c(u)^2. \tag{A.11b}
\]
Thus
\[
\frac{\partial u}{\partial k_1} = \frac{\eta c'(u)^2 c(u)}{([-\eta c'(u)^2 + c(u)c''(u)][k_2(\eta - 1) - k_1c'(u)] - \eta k_1 c(u)c'(u)c''(u))}. \tag{A.12}
\]
To find \(\partial p / \partial k_1\) we take the partial derivative of \(p\) in (A.5) with respect to \(k_1\), we obtain
\[
\frac{\partial p}{\partial k_1} = \frac{\eta k_2 c'(u)c'(u)}{[k_2(\eta - 1) - k_1c'(u)]^2}. \tag{A.13}
\]
Considering now (A.12), (A.13), (A.9), we obtain
\[
\text{Sign} \left[ \frac{dp}{dk_1} \right] = \text{Sign}[-[(\eta - 1)k_2 - k_1c'(u)]\eta c'(u)^2
+ [(\eta - 1)k_2 - (\eta + 1)k_1c'(u)] \times c(u)c''(u)]. \tag{A.14}
\]
Thus, considering (A.7), a necessary condition that \(\text{Sign}[\partial p / \partial k_1] = +1\), is that
\(\eta - 1)k_2 - k_1(\eta + 1)c'(u) > 0\).

**Part (ii)**: Similarly as in Part (i), we consider (3.15) and take the derivative (total) of \(p\) with respect to \(k_2\) we obtain
\[
\frac{dp}{dk_2} = p'(u) \frac{\partial u}{\partial k_2} + \frac{\partial p}{\partial k_2}. \tag{A.15}
\]
where \(p'(u)\) is as in (A.6) and \(\partial u / \partial k_2\) and \(\partial p / \partial k_2\) are the partial derivatives of \(u\) and \(p\) with respect to \(k_2\).

To find \(\partial u / \partial k_2\) we take this time the derivative of the implicit function \(g(u, k_1, k_2, x)\) with respect to \(k_2\) we obtain
\[
\frac{\partial g}{\partial u} \frac{\partial u}{\partial k_2} + \frac{\partial g}{\partial k_2} = 0.
\]
Thus,
\[
\frac{\partial u}{\partial k_2} = -\frac{\partial g}{\partial k_2} \frac{\partial g}{\partial u}.
\]
Considering (A.10) we obtain
\[
\frac{\partial g}{\partial k_2} = \frac{ak_1 c'(u)^2(\eta k_2 c(u))^{1-\eta}}{k_2(\eta - 1) - k_1c'(u)}
- \frac{\eta k_2 c'(u)c'(u)c''(u)}{k_2(\eta - 1) - k_1c'(u)}. \tag{A.16}
\]
and considering (A.11b) again and (A.10) we have
\[
\frac{\partial u}{\partial k_2} = \frac{\partial u}{\partial k_1} \left(\frac{(\eta - 1)k_2 - (\eta + 1)k_1c'(u)}{\eta k_2 c'(u)}\right). \tag{A.11b}
\]
To find \(\partial p / \partial k_2\) we take the partial derivative of \(p\) in (A.5) with respect to \(k_2\), we obtain
\[
\frac{\partial p}{\partial k_2} = \frac{\eta k_1 c'(u)c'(u)}{[k_2(\eta - 1) - k_1c'(u)]^2}. \tag{A.17}
\]
Considering now (A.16), (A.12), (A.17), (A.15) we obtain
\[
\text{Sign} \left[ \frac{dp}{dk_2} \right] = \text{Sign}[-[(\eta - 1)k_2 - k_1c'(u)]\eta c'(u)^2
+ [(\eta - 1)k_2 - (\eta + 1)k_1c'(u)] c(u)c''(u)]
= \text{Sign} \left[ \frac{dp}{dk_1} \right]. \tag{A.18}
\]
Thus, again considering (A.7), a necessary condition but not sufficient that \(\text{Sign}[\partial p / \partial k_2] = +1\), is that
\(\eta - 1)k_2 - k_1(\eta + 1)c'(u) > 0\). \(\square\)
References


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